Absorptive capacity, R&D subsidies and growth

A thesis submitted in fulfilment of the requirements for the degree of Master of Social Sciences in Economics

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Tiivistelmä Referat - **Abstract**

I have introduced the notion of *absorptive capacity* into an innovation-driven growth model. The model features firms with heterogeneous size and innovation capability. The economy's aggregate output growth rate is driven by the growth of the productivity index of firm's intermediate goods. There are firms already operating at least one active product line and potential entrants not owning a product line, but engaged in research in order to innovate and enter the economy. Each firm engages in R&D activities to improve upon existing intermediate good or to discover a completely new one. An incumbent firm may exit the economy exogenously and more importantly endogenously due to creative destruction or due to obsolescence. The consideration of the firm's absorptive capacity in Cohen and Levinthal's sense shaped the specifications of the innovation production functions for entrants and incumbents. I have set a proxy for the firm's absorptive capacity that considers the quality level of active product lines owned by the firm, their number and their closeness. Then, incumbent firm's absorptive capacity confers to the it an innovation efficiency advantage compared to potential entrant, in improving upon its own product lines or in innovating in a product line not existing in its portfolio. For potential entrants, the average quality level of intermediate good in the economy acts as an *extra difficulty* they must overcome since the did not participate in building that economy's average level of quality, and lack then the absorptive capacity that an average incumbent firm in the economy would possess. Although the general structure of the equilibrium growth rate obtained is the same as in the basis model (Acemoglu et al., 2013), however, the content of incumbents' rate of innovation and the rate of entry for potential entrants is different and more likely to deliver different values in equilibrium. This framework may be useful in studying empirically the effects of R&D subsidies on economic growth and lead to different results obtained by the main reference article, that found that subsidies to incumbents' R&D are pervasive for the growth of the economy.

Avainsanat-Nyckelord-Keywords
Innovation, Absorptive capacity, R&D subsidies, Selection

Säilytyspaikka-Förvaringställe- Where deposited

Muita tietoja-Övriga uppgifter-Additional information

Contents

List of Abbreviations

CRRA	**C**onstant **R**elative **R**isk **A**version
BGP	**B**alanced **G**rowth **P**ath
GDP	**G**ross **D**omestic **P**roduct
LHS	**L**eft **H**and **S**ide
RHS	**R**ight **H**and **S**ide
FOC	**F**irst **O**rder **C**ondition
R&D	**R**esearch and **D**evelopment
EU	**E**uropean **U**nion
LBD	**L**ongitudinal **B**usiness **D**atabase
CMF	**C**ensus of **M**anufacturers
NBER	**N**ational **B**ureau of **E**conomic **R**esearch
US	**U**nited **S**tates of America
USPTO	**U**nited **S**tates **P**atent and **T**rademark **O**ffice
SMM	**S**imulated **M**ethod of **M**oments
Exp	**E**xponential function
ln	**N**eperian **L**ogarithm function

List of Symbols

$C(t)$	Consumption aggregate at time *t*
$c_j(t)$	Consumption of good j at time *t*
U_0	Present value of the utility from lifetime consumption
ϑ	Rate of time preference
ρ	Discount rate
ϵ	Elasticity of substitution between products
N_t	Set of active product lines at time *t*
$Y(t)$	Aggregate output at time *t*
L^P	Total demand of unskilled labour
L^F	Skilled labour demand for operations
L^{RD}	Skilled labour demand for R&D activities
L^S	Total supply of skilled labour
$r(t)$	Equilibrium interest rate on assets
$w^u(t)$	Unskilled wage rate
w^s	Skilled wage rate
q_j	Productivity of intermediate good *j*
$W(t)$	Total household's asset at time *t*
$I(t)$	Total household's income at time *t* $\hat{q}_j$
l_{jj}	Number of workers employed by firm f to produce the good *j*
θ^H	High-type firm
θ^L	Low-type firm
α	Probability of having high-type firm in the economy
Q	Productivity index of the economy
n_f	Number of product lines owned by firm *f*
X_f	Product innovation flow rate for firm *f*
x	Innovation intensity
$\bar{q}_j$	Average productivity of intermediate goods owned by firm *f*
$\bar{q}_{np}$	Average productivity of inactive product lines
λ	Innovation size factor
τ	Creative destruction rate
φ	Exogenous destructive shock
ν	Transition rate from high-type to low-type
$\hat{q}_{l,min}$	Relative productivity threshold for obsolescence in a low-type firm
$\hat{q}_{h,min}$	Relative productivity threshold for obsolescence in a high-type firm
Φ^h	Share of active product lines owned by high-type firms
Φ^l	Share of active product lines owned by low-type firms
Φ^{np}	Share of inactive product lines
$\tilde{V}_l(\tilde{Q})$	Stationary equilibrium value function for a low-type firm
$\tilde{V}_h(\tilde{Q})$	Stationary equilibrium value function for a high-type firm
ϱ	Non-R&D related positive productivity shock

$\tilde{\pi}(\hat{q})$	Operating profit from product line of relative productivity $\hat{q}$
Υ^h	Franchise value of a product line of relative productivity $\hat{q}$ of high-type firm
Υ^l	Franchise value of a product line of relative productivity $\hat{q}$ of low-type firm
ϕ	Amount of skilled labour requirement for operation tasks
$F_h(\hat{q})$	Stationary equilibrium productivity distribution high-type product lines
$F_l(\hat{q})$	Stationary equilibrium productivity distribution of low-type product lines
$F(\hat{q})$	Stationary equilibrium productivity distribution for inactive product lines
$f_k(\hat{q})$	Density function for $F(\hat{q})$
g^h	Growth rate of the productivity index of product lines owned by high-type firms
$\bar{q}_{ft}$	Firm's f proxy of absorptive capacity at time t
g^l	Growth rate of the productivity index of product lines owned by low-type firms
g	Growth rate of the productivity index of the whole economy
$\bar{q}$	Average productivity level in the economy
Ω^k	The option value for k-type firm
$\tilde{Q}_t^k$	Productivity index of k-type firms

Chapter 1

Introduction

The European Commission under its *Framework Programme for Research and Innovation, Horizon 2020* [1] promises to support firms' R&D activities within the Union for €80 Billions in the form of grants by 2020. Similar framework programs have been implemented since the years 80's providing during the years 90's up to 2 billions US $ per year (European Commission). Besides the EU framework, most governments dispose their own national subsidies intervention set-up. Several types of market failures have been presented as argument justifying such government interventions: knowledge spillovers (Grossman and Helpman, 1991), making social returns to R&D higher than private returns, firms' low opportunity cost in severe crisis (Ewijk, 1997), capital market imperfections (Stiglitz, 1993) and the *infant industry* argument (Klepper, Moen, and Griliches, 2000). In practice, these funding may be allocated as bailout, as grant covering the whole cost of the project without any own R&D investment contribution requirement for the receiver private firm or as an additional R&D investment aiming for stimulating private R&D investment.

Although the effects of public subsidies of R&D on private receivers' R&D spending and more importantly their performance may be well known and almost unanimously acknowledged as been positive (Klepper, Moen, and Griliches, 2000, Clausen, 2009, Gorg and Strobl, 2007, Hall and Reenen, 1999), what effects such subsidies may have on the long-term growth rate of an economy remain ambiguous.

On the one hand, Segerstrom, 2000 in an earlier paper found that the long-run effects of R&D subsidies on growth depends on a set of parameters. Given the specification of his model, although R&D subsidies appear to enhance growth in the short-run, the growth-retarding outcome is realized for a wide range of plausible parameter values, at a temporal horizon depending on how fast convergence occurs in the model.

On the other hand, (Acemoglu et al., 2013) found that subsidies to incumbents' R&D reduce growth and welfare whereas subsidies to both entrants and incumbents' R&D combined with very high and almost impossible tax rates on incumbents' operations will improve growth and welfare. The authors pointed out a strong negative selection effect as the aggregate growth reducing mechanism from R&D subsidies: R&D subsidies to incumbents impede the reallocation of R&D inputs from inefficient incumbents to more efficient entrants, altering then the productivity-survival link within industry.

[1] Horizon 2020, The Framework for Research and Innovation, COM(2011) 808 Final. Brussels, 30.11.2011

Considering the latter findings, past R&D subsidies to private firms might explain part of the European economic slowdown these last decades on the one hand and these findings might stand as a warning for certain inappropriateness of using subsidies to R&D as stimulus for the industrialization of backwards economies. However, these findings may only not sound counter-intuitive given subsidies to research have been proved to increase receivers firms investment in R&D [2], but most importantly are at odds with recent findings from Acemoglu and Cao (2015) stating the bulk of economy's productivity growth stemming from incumbents and then less significance to reallocation-driven productivity growth [3]. This divergence indicates that the theoretical framework for analysing the effects of R&D subsidies on growth should be revisited.

The aim of this study is to provide a theoretical framework useful to have more insights on the long-run effects of R&D subsidies on the economy's aggregate growth, departing from the firm level and taking into account the determinants of innovation success and the industry dynamics affecting aggregate growth that such policy may trigger.

I built a theoretical model based upon Acemoglu et al. (2013) in which there are two sets of firms: Incumbents and potential entrants. Incumbents operate at least one active product line and are engaged in research activities. Potential entrants do not own a product line but are engaged in R&D activities in order to innovate and enter the industry. As entry, exit is endogenous. Besides the heterogeneity in size for incumbents, both incumbents and entrants are heterogeneous in terms of their innovation capability. I introduced into the analysis, the concept of firm's *absorptive capacity* defined as the firm's ability to identify, assimilate and exploit knowledge from external environment (Cohen and Levinthal, 1989) and based upon its past R&D activities.

In fact, recent empirical works found that ownproducts improvements by incumbents appear to be by far larger than creative destruction by both entrants, and that incumbents mostly improve upon their own products rather than other incumbent's products (GarciaMarcia, Hsieh, and Klenow, 2015). Similarly, Akcigit and Kerr (2015) demonstrated that higher rate of radical inventions of small firms (mostly new entrants) is an outcome of their R&D investments choices instead of their higher capabilities in research [4]. These findings combined with Cohen and Levinthal's insights seem to be very suggestive in highlighting how critical is the firm's *absorptive capacity*in increasing its innovation capabilities.

Given the determinants of such a construct pointed out by previous literature (Cohen and Levinthal, 1989, Cohen and Levinthal, 1990b, Nieto and Quevedo,

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[2]Manfield (1986), Clausen (2009), Gorg and Strobl (2007), Lach (2000), Hall and Reenen (1999), Einiö (2014)

[3]Acemoglu et al. (2013)'s work, based on previous literature (Foster, Waltiwanger, and Krizan (2001) and Foster, Waltiwanger, and Krizan (2006)) stressing on the important contribution of reallocation (70-80% including mainly entry of more efficient and exit of less efficient firms) in the economy's productivity growth, found that entrants contribute to 58% of the productivity growth of the US economy whereas later on Acemoglu and Cao (2015) using different theoretical framework found entrants' contribution at the firm level less than 35%

[4]They provided empirical evidence with data from LBD, USPTO data and NBER patent database, based on the specification of their general equilibrium model departing from firm level differentiating internal research from external research

2005) and as suggested by Aghion and Jaravel (2015), I assume any incumbent having advantage of it over any potential entrant.

I set up a simple proxy for firm's absorptive capacity given by the average quality of product lines the firm owns and defined new different specifications for incumbents and entrants' innovation functions. An incumbent's absorptive capacity grants the firm an advantage in innovation capability and the economy's average quality level of product lines acts as *extra difficulty* to overcome for potential entrants.

The model delivers identical growth rate of the economy's in terms of structure, however with different content for the rate of innovation by incumbent firms and the entry rate by potential entrants. It is likely that with the introduction of the notion of absorptive capacity into the reference framework, the calibration of my obtained model using the same data as (Acemoglu et al., 2013) would lead to different values of innovation rate by both sets of firms and then different effects of R&D subsidies on growth on the one hand and different relative contribution of incumbent firms to the aggregate growth rate of the economy on the other hand.

For the following of my study, I will start by exploring the previous literature related to the research question including the relevance of the construct of absorptive capacity into my analysis framework, then will follow the extended Acemoglu et al. (2013)'s theoretical model followed by a discussion on the expected results and finally, I will proceed to concluding remarks.

Chapter 2

Literature Review

My research question is related to several previous investigations in the economic literature. First, the construct of absorptive capacity and R&D spillovers within the context of innovative activities. Second, resources reallocation and productivity-survival link within an industry with firm heterogeneity. Finally and most directly to the effects of R&D subsidies on growth.

2.1 R&D, Spillovers and absorptive capacity

Concerning the relationship spillovers-innovation, its study can go back to Isaac Newton. "If I have seen further, it is by standing on the shoulders of the giants". The first point this statement from Sir Isaac Newton[1] reveals is the importance of existing breakthroughts upon which researchers rely on in order to build new ones. Many studies have supported this statement from Newton. An empirical study by Caballero and Jaffe (1993) assessing knowledge spillovers and built upon patent registrations and citations suggested that private research productivity declines of the same size as spillovers diffusion declines. Lin (2015) found that knowledge spillovers from research universities affects positively firm's growth. However, the same spillovers from R&D are pointed out as undermining firms incentives to invest in innovative activities (Grossman and Helpman (1991), Caballero and Jaffe, 1993) and justify government interventions. Although it is clear that spillovers affect positively research productivity, it is still a twist to argue that they undermine incentives to innovate. In fact, as stated by Klepper, Moen, and Griliches, 2000, given the complementary relationship that exists between firms' R&D spillovers, firms have incentives to invest in order to be able to capture other firms spillovers: the effects of spillovers on R&D incentives may then be ambiguous. A firm conducting only imitative R&D will contribute little to the spillovers pool, whereas benefiting more from firms conducting innovative R&D and in this case, spillovers may undermine incentives to innovate of the latter.

The second and less explicit point that may come up from the above citation from Sir Isaac Newton is that the conditional ability to grasp upon the *Giant's body* should not be underestimated. Although the importance of previous breakthroughs from external sources is explicitly pointed out in this statement, another implicit condition is the ability for new researchers to grasp giant's body and reach his shoulders before standing upon them. This calls then on firm's ability to benefit from available external knowledge. Cohen and Levinthal, 1989

[1] Sir Isaac Newton, letter to Robert Hook, February 5, 1675

pioneering that ability, coined the construct of *absorptive capacity* and defined it as the firm's ability to identify, assimilate and exploit new external knowledge. Although this definition does not reveal explicitly the content of the construct, it positions it as a firm's asset or a firm's resource. Later on in *Fortune favors the Prepared Firms* (Cohen and Levinthal, 1990a), Cohen and Levinthal argued the firm's absorptive capacity is not only the capability that enables firms to exploit new external knowledge, but also allows it to predict accurately the nature of the future technological advancement. Furthermore, the absorptive capacity increases the speed and frequency of firms innovation because such innovations primarily draw on firms knowledge base (Kim and Kogut, 1996).

At this point, we may think of absorptive capacity as enabling capability in seizing technological opportunities only in domains related to prior firms knowledge, then not affecting positively radical innovations. Bosch, Volberda, and Boer (1996) argued that although incremental innovation are supported by an absorptive capacity that provides a deep understanding of narrow range of closely related domains and helps to increase that depth, radical innovations may also be supported by an absorptive capacity based on a broad range of loosely related knowledge domains. Precisely, the firms absorptive capacity is built upon its prior knowledge base and is a byproduct of the firms R&D investments (Cohen and Levinthal, 1990b). Later on, Hurry, Miller, and Bowman (1992) argued that the more an organization innovates in an area, the faster it increases its absorptive capacity in that area. This point then suggests a recursive relationship between absorptive capacity and innovation. Innovation as a positive outcome of a new learning process fuelled by absorptive capacity, reinforcing in turn this latter. The sociocognitive processes underlying the concept of absorptive capacity as suggested by Cohen and Levinthal, point out the firms absorptive capacity as depending on the one of its individuals and more importantly to view it not only from a structural perspective, but from a dynamic perspective. As consequence, absorptive capacity is more likely to be developed and maintained as a byproduct of routine activities when the knowledge domain that the firm is committed to exploit is related to its current knowledge base, otherwise, separated and exclusive effort must be dedicated for its creation (Cohen and Levinthal, 1990b)

The firm's absorptive capacity is then a capability that is built over time upon firms prior knowledge base from past R&D activities. However, this argument may seem not to clearly favour an incumbent over a potential entrant, since both are conducting R&D activities. Not only it is more likely an incumbent would have spent more time and resources on R&D than an outsider, it may also possesses an advantage from its production experience (Klepper, 1996) and increases its absorptive capacity. Empirical investigations on the effects of absorptive capacity on innovation outcomes have been carried out. Cohen and Klepper (1996) via a firms' survey, noticed that innovative output depends positively on research input, or research intensity or innovative effort which in turn does not hinges on firm's size (Klette and Kortum, 2004). Nieto and Quevedo (2005) pointed out the differences in absorptive capacity across firms as driving these differences in R&D intensity across firms. The tacitness of the absorptive capacity emphasized by Cohen and Levinthal (1990b) and this conclusion shed then the light on the persistence in time in R&D intensity difference. Klette and Kortum (2004) acknowledged that past knowledge affects positively R&D intensity, but is offset by diminishing returns on expanding R&D investment whereas Nieto and Quevedo (2005) considers a broad construct of absorptive capacity, including past knowledge.

Given the above mentioned literature, the firm's absorptive capacity turns out to be a critical input for its innovation production function.

2.2 Industry dynamics

Regarding the industry dynamics with respect to R&D activities, many scholars have paid particular attention to it. Jovanovic (1982), Cin, Kim, and Vonortas (2013) and Nocke (2003) pointed out the firm-level productivity-survival link underlying reallocation as driver of productivity growth at the aggregate level. Klepper (1996), with a firm-level endogenous technological change model featuring firm's heterogeneity in random innovative capabilities and size, studied the patterns of entry, exit, market structure and innovation of technologically progressive industry from its birth to its maturity. His model predicts that over time, firms devote more effort to innovation, but the number of firms and the rate and diversity of product innovation eventually wither. In mature industry, incumbent possessing the advantage of size focus on improving production processes, increasing then further the minimum efficient size of firm and new firms can enter the market only in succeeding in radical innovations (Klepper, 1996). In Klepper (1996)' model, firm's expertise in product innovation differs randomly, no matter whether an entrant or an incumbent. The type of distinctive capability affecting the type of innovation (product or process) in which a firm engages on is randomly distributed. However, though for product innovation, the type of capability manifests itself by the type of client the firm services, the author acknowledged that capability in process innovation is based on information that firms commonly generate through production. This points out an advantage an incumbent as an early entrant would have over a newly entrant in surviving the industry's shakeout . With a different approach,

Ewijk (1997), focusing on industry dynamics along the business cycles found that less productive firms are those that suffer more in recession; however if taking into account the *substitution effect*[2] that may take place in that phase of the cycle, there is hope for technology improvement. However, this substitution-effect-driven productivity may be offset by the *rusting effect* [3]. In fact, individual firm's growth rate is greater in recession than in boom (substitution effect:positive effect) but the aggregate level of inactivity (rusting effect due to firm exit:negative effect) may cause productivity loss. The author suggests that the former effect prevails in mild fluctuations whereas the later in severe fluctuations. Given the fact that most of entry is in boom phase (Ewijk, 1997), this result may suggest

[2]Substitution of production activities by R&D activities during the recession phase of the business cycle (Aghion and Saint-Paul, 1998)

[3]The loss of knowledge associated with firms exit: Caballero and Hammour (1991)

a ground for preventing exit of incumbents in severe crisis (Rationale for R&D subsidies).

Foster, Haltiwanger, and Syverson (2008) using micro-data from firms, focused on investigating the contribution of plant-level technology and demand fundamentals to survival and selection-based productivity growth. They found that exiting firms have lower productivity levels (either revenue based or physical-quantity-based) than incumbent. We may then think of Acemoglu et al. (2013) main finding, that is the alteration of efficiency-based selection by R&D subsidies, as being in line with Foster, Haltiwanger, and Syverson (2008)'s result. However, the gap is larger in magnitude for revenue based productivity:demand-side shocks effects significantly affect firms growth and dominantly determine firm's survival (Foster, Haltiwanger, and Syverson, 2008). This means within the same industry, a firm may be obliged to exit while having higher physical quantity productivity than another firms remaining in the industry. In other words, reallocation within industry may be driven by profitability and not real output productivity. Then, reallocation within industry may then not automatically drive aggregate productivity growth. Which means a business may exit while being able to contribute more than a surviving incumbent to the aggregate productivity growth within an industry. Although entrants have higher productivity growth than incumbent (Foster, Haltiwanger, and Syverson, 2008), their entry rate may be inversely correlated with productivity growth since depressing profitability of incumbents innovation (Acemoglu and Cao, 2015).

Another recent contribution in the reallocation- productivity growth literature is from Acemoglu and Cao (2015). Using data from US innovative firms, with a theoretical framework in which both entrants and incumbents are carrying out innovative activities, they found that the large fraction of within industry productivity growth is accounted by continuing establishments, which is at odds with findings from previous literature on technological change models.

From the above mentioned literature, resources reallocation is driver of productivity growth and the within industry survival is not always productivity-based.

2.3 R&D subsidies and growth

The first wave of research in this section is related to the question of effectiveness of government incentives on firm investments in R&D, and the second on the effects of these subsidies on the long term growth of the economy.

The question of the effectiveness of incentive measures to R&D investment, pioneered by Manfield (1986) who using data from US innovative firms, found that there was less than *1.8* per cent increase in R&D expenditure due to tax credit and that the due increase in R&D expenditure was at most a third of the foregone revenue by the government. He then called for changes in tax credit and the way to incentivize effectively firms for R&D investments. In the same line,Clausen (2009), using data from a set of Norwegian firms conducting R&D activities, found that R&D subsidies to *Research* activities stimulate R&D spending within firms whereas *Development* subsidies substitute it.

Later on, Gorg and Strobl (2007) exploiting data from a sample of manufacturing firms in Ireland, found that both the size of public subsidy and the nature of the firm matter for the effects of public R&D on private R&D : small grants increase R&D spending for domestic plants whereas large grants lead to crowding-out effects. In contrast, grants provision causes neither additionality nor crowding out effects for foreign plants no matter the size of the grant. Both Lach (2000) and Hall and Reenen (1999) found positive effect of R&D subsidies on receiving firms' R&D investments. Regarding the effect of R&D subsidies on receiving firm's productivity, Einiö (2014), exploiting geographic variation in government funding based on the ERDF[4] population-density rule found a positive effect of R&D subsidies on long run productivity for marginal firms that received subsidies resulting only from increase in governmental support in their region. This result does not tell more about average effect of R&D subsidies on firm's productivity. Furthermore, given granting agency's criterion, those marginal firms turned out to be those having lower return rate of R&D, then lower research productivity. Considering this characteristic, the increase in productivity three years latter on in average turns out to be in line with Cohen and Levinthal (1989)' view of R&D activities. In fact we may think that the support allowed them the first here to conduct research, which increased their absorptive capacity gradually making them more productive in research afterwards and with results on production productivity the third year.

Regarding the effects of R&D subsidies on growth, the research was pioneered by Davidson and Sergerstrom (1998). Considering decreasing returns to R&D activities in an endogenous growth model featuring both innovation and imitation, they found that innovative R&D subsidies foster growth whereas imitative R&D subsidies lower it. Furthermore, for such an economy, the effects of general R&D subsidies on the growth rate proxied by consumers utility growth depends on the relative equilibrium value of innovative/imitative R&D ratio. Using computer stimulation, they found that optimal public policy will consist in heavy subsidies to innovative R&D and heavy tax rate on imitative R&D. However, besides the fact that not taking into account the spillover effects of research, their model did not allow for entry and exit of firms, then not capturing the effects of research resources reallocation on growth due to subsidies policy.

Later on, Segerstrom (2000) in an endogenous growth model featuring both vertical and horizontal R&D as engines of growth found that the long-run effects of R&D subsidies on growth depends on a set of parameters. General R&D subsidy enhances long-run growth if it promotes the stronger of the two engines of growth in the economy. Otherwise, if it promotes the weaker engine, it will retard growth in the long-run. According to Segerstrom, an engine is stronger when it is subject to lower diminishing returns compared to the other engine. Given the properties of the model, R&D subsidies always appear to enhance growth in the short-run and the temporal horizon at which the negative effects of R&D subsidies will start affecting the economy will depend on how fast convergence occurs in the model. Economic conditions in which either of the engines of innovation has lower marginal returns remain not clearly stated. However,

[4] European union Regional Development Funds

Segerstrom (2000)'s model did not allow the margin for entry and exit, then not being a suitable framework for analysing selection effects and industry structure transformation that such policies may trigger.

Finally, Acemoglu et al. (2013) found that subsidies to incumbent firms' R&D retard growth. In fact, although R&D subsidies improve performance of receiving firms, there may exist some counter effect mechanisms by which they affect negatively both the overall growth rate of the economy and moreover, welfare.Acemoglu et al., 2013 pointed out a strong negative selection effect as one of those growth reducing mechanisms from R&D subsidies. The authors built a micro founded model of endogenous growth with innovation by incumbents and entrants. Using data from a sample of US innovative firms from the NBER patents database, the USPTO database, the LBD and the CMF database, they estimated the parameters of the model and found that the economy growth rate is reduced when incumbents' R&D or operations are subsidized. The mentioned strong negative selection effect is due to the fact that R&D subsidies to incumbent firms hinders the entry of more efficient new firms and the exit of less efficient incumbents, and then prevent an optimal reallocation of R&D inputs (skilled workers). Their policy experiments suggested an industrial policy favouring entry of more efficient new firms while spurring exit of less efficient incumbents, by coupling heavy tax on incumbents operations and small subsidies to their R&D.

Then, the ambiguous and somewhat diverging findings from previous literature highlight the uncertainty that may surround the effects of R&D subsidies on the aggregate growth rate of the economy.

Chapter 3

A theoretical model

In this section, my aim is to set up a theoretical framework based on Acemoglu et al. (2013) embedding features allowing the study of the effects of some policies targeting firms on the growth of the whole economy, specifically the effects of R&D subsidies on the long-run economic growth.

First, the framework links firm-level productivity growth to aggregate growth. Secondly, it encompasses several level of heterogeneities: innovation capacities, R&D effort and size. Third, it allows for both quality improvement and variety expansion both for incumbents and potential entrants. Finally, besides exogenous exit, it incorporates a mechanism for endogenous exit and entry, the survival of the firm in both cases being productivity-based in absence of any support policy. These included features aim at capturing some stated empirical regularities and allowing for effects of targeted firm-level policy measures.

Unlike in Acemoglu and Cao (2015), in my model, incumbent and potential entrant are equally likely to invest in new product exploration as in Klette and Kortum (2004) and Acemoglu et al. (2013) , but for the same research intensity, an incumbent with only one active product line is not equally likely as a potential entrant to succeed, incumbent having more likelihood, thanks to its *absorptive capacity*. The model starts by specifying the individual firm‘s innovation behaviour either incumbent or potential entrant and leads to a general equilibrium accounting for the aggregate growth rate as stemming from any discovery made at the firm-level.

3.1 Preferences and final good

A representative household in this continuous time economy has the CRRA preferences given by

$$U_0 = \int_0^{\infty} exp^{(-\rho t)} \frac{C(t)^{1-\vartheta} - 1}{1-\vartheta} dx \tag{3.1}$$

where $\rho > 0$ is the the discount factor and $C(t)$ a consumption aggregate. The consumption aggregate is given by

$$C(t) = \left(\int_{N(t)} c_j(t)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{1}{\epsilon-1}} \tag{3.2}$$

where $c_j(t)$ is the consumption of product *j* at time t. $\epsilon > 0$ is the elasticity of substitution between products and *N(t)* is the set of active products lines at time

t. Because R&D costs are in terms of labour only, the output of each product line equals the consumption of that given product and the following equality holds:

$$c_j(t) = y_j(t)$$

$y_j(t)$ being the amount of product j produced at time t in the economy. This implies that the aggregate output of the economy is equal to the aggregate consumption. We have then,

$$Y(t) = C(t)$$

In the economy, there are two types of labour: skilled labour and unskilled labour. Unskilled labour which fixed total supply from the representative household is normalized at *1*, is used in production exclusively. Meanwhile, skilled workers are used within the R&D sector and also used for operations like management, back office functions, and other skilled-required tasks. It is assumed that the operations of each product line and of each entrant as well require $\phi > 0$ units of skilled labour.

The fixed skilled labour supply from the representative household is of measure L^S. The total labour demand for unskilled labour, for skilled labour in operation duties and for R&D activities are respectively L^P, L^F and L^{RD}. The labour market clearing will then require the following conditions:

$$L^P = 1$$

and

$$L^F + L^{RD} = L^S$$

The optimization problem of the representative household will consist in maximizing (3.1) subject to its following flow budget constraint

$$\dot{W}(t) + C(t) \leq r(t)W(t) + I(t)$$

and the usual noPonzi condition. $I(t)$ the total labour income, is given by

$$I(t) = w^u(t) + w^s(t)L^S$$

The asset position of the representative household $W(t)$ is given by

$$W(t) = \int_{N(t)} V_j(t)dj.$$

$r(t)$ is the equilibrium interest rate on assets and $w^u(t)$ and $w^s(t)$ are respectively the unskilled and skilled wage rates.

3.2 Intermediate good production

Intermediate goods constitute the point from which the growth of the economy stem, and is therefore the target of any R&D activity. An improved intermediate good produces in a more efficient way. A new intermediate good expands the range of production tools, the variety of products that will be produced in the economy and increases the range of choices available to consumers. Each intermediate good is for a production of a single product line and thereafter, product line or good and intermediate good are interchangeably used. Assuming that there is enforceability of intellectual property rights, at a time t, there is only one producer, a monopolist for any given intermediate good j. This monopolist has the most advanced technology for that given intermediate good. There are two different sets of firms in this economy: A set of active firms ($\mathcal{F}$) which has the monopoly over at least one product line and a set of potential entrants conducting research for innovation. Any incumbent or even potential entrant, since conducting R&D activities, may end up in innovating upon any existing product line and becomes the new monopolist for that product line. For producing that leading-edge intermediate good j of productivity level $q_{f,j}$,the monopolist needs ϕ units of skilled labour for tasks like management and other operations, and hires $l_{f,j}$ number of unskilled labour. It then has access to a linear technology in the form

$$y_{f,j} = q_{f,j} l_{f,j} \tag{3.3}$$

with $y_{f,j}$ being the output for that good j. From this specification (3.3),the marginal productivity of unskilled labour is $q_{f,j}$ units and then the marginal cost of production is then given by $\frac{w^u}{q_j}$.

Therefore, the variation of unskilled wage affects marginal cost of production and then product lines activity decisions. I define then for any product line j, its relative productivity as

$$\hat{q} \equiv \frac{q}{w_u} \tag{3.4}$$

This relative productivity may evolve either as outcome of innovation upon the given product line or following variation in the unskilled wage rate.

This model economy is then made up of active firms of set $\mathcal{F}$, each firm operating at least one product line, which product lines are imperfect substitutes. Then, the economy productivity index is defined as

$$Q \equiv \left(\int_{\mathcal{N}} q_j^{\epsilon-1} dj \right)^{\frac{1}{\epsilon-1}} \tag{3.5}$$

This index gives the average level of the quality of the production tool in the economy and evolves according to R&D outcomes.

3.3 Heterogeneity and industry dynamics

Firms in my model economy are heterogeneous. First, firms are of different sizes: the size here is proxied by the number of product lines that the firm operates. An incumbent firm has at least one active product line. Second, firms of different age coexist: firms newly entered and old firms that survived from creative destruction and obsolescence. Third, firms are different in *type*: the firm's type refers to its comparative innovation capacity. Any firm in the economy can be either of high-type (θ^H) or low-type (θ^L), according to its innovation capacity. This difference is assumed to be marked between two firms in the economy, when holding other aspects of heterogeneity constant. For example, from two incumbent firms, one may be of high-type and the other of low-type, idem for two potential entrants or two firms of the same age. The type here refers to any other capability that a given firm either potential entrant or incumbent may have in innovating that has not been gained from past research activities.

Concerning the absorptive capacity, I assume that any incumbent firm is endowed with more absorptive capacity than any potential entrant, regardless of its type. This constitutes the main divergence point from Acemoglu et al. (2013)'s framework. The combination of firm's type and firm's absorptive capacity impacts innovation as described below, for incumbents and potential entrants.

3.3.1 Innovation by incumbents

I start this section by examining the R&D specification used by the reference model, Acemoglu et al. (2013)[1] following Klette and Kortum (2004) based on the notion of *knowledge capital*. Although this notion of knowledge capital can be seen as capturing the same content as Cohen and Levinthal (1989)'s concept of *absorptive capacity*, it leads to an innovation flow rate specification for both incumbents and entrants presenting some counter intuitive shortcomings.

First, it proxies the firm's *knowledge capital* by the number of product lines it operates. Although it is simple, it however gives the same weight to two firms with the same number of plants or product lines no matter whether one is at the bottom of the equilibrium productivity distribution whereas another is at the top of the productivity distribution.

Second, it confers the same amount of knowledge capital to an incumbent operating one product line and a potential entrant, which is counter-intuitive given the comparative advantage of past research experience that any incumbent benefits. By then, in an environment with knowledge spillovers, via the frontier research dynamics either in technological space or market space, this notion of *knowledge capital* is then limited in seizing properly the capabilities that innovative firms would have built via their past R&D activities and which seem critical in explaining at least part of heterogeneities in innovation intensity across firms (Cohen and Levinthal (1989), Cohen and Levinthal (1990b), Nieto and Quevedo (2005)). Therefore, considering the notion of firm's absorptive capacity in Cohen and Levinthal's sense, a different proxy of it entering the innovation function

[1] For an incumbent with n product lines, of type θ^K and h skilled workers hired for research, it adds one new product in its portfolio at the following rate: $X_f = \theta_f^{\gamma} n_f^{\gamma} h_f^{1-\gamma}$

should be proposed.
In order to model the innovation production functions, I first state the following assumptions:

Assumption 1: Research is undirected across product lines.
Firms do not know ex-ante upon which product line they will innovate, then their expected returns to R&D is the expected value across all product lines $j \in [0, 1]$

Assumption 2: Within a given incumbent firm, the absorptive capacity as a knowledge resource is non-rival.
The use of the firm's absorptive capacity as input in one product line research unit does not diminish the quantity available to other research units within the same firm. Absorptive capacity as innovation input, enters the firm's per product innovation rate function at the same amount as in the firm's total flow rate of innovation.

I then model the innovation by an incumbent firm as follows:

Incumbent firm can engage in improving upon its own product or improving upon another firm product or even to innovate upon an inactive product line. For an incumbent firm f of type $k \in \{h, l\}$, in time t, with h skilled workers hired for research activities, and operating n_f product lines with each of them j having a productivity q_j adds one more product line in its portfolio at the following flow rate:

$$X_{ft} = \theta_f^{\gamma} \bar{q}_{ft}^{\gamma} (h_{ft})^{1-\gamma} \tag{3.6}$$

[2]

with

$$\bar{q}_{ft} \equiv \left(\int_{n_{ft}} q_{jf}^{\epsilon - 1} dj \right)^{\frac{1}{\epsilon - 1}} \tag{3.7}$$

$\bar{q}_{ft}$ is my proxy of the firm's f absorptive capacity which is a sort of productivity weighted index across firm f's operated product lines. I assume $\bar{q}_f > 1$ in equilibrium. Here it acts as a capability the firm's has gained from its past R&D activities. This specification supposes that firms' past research activities grant them more efficiency in innovating at the current period.

On top of avoiding the above mentioned shortcoming for Acemoglu et al. (2013)'s use of *knowledge capital*, my use of *absorptive capacity* and its proxy also takes into account the fact that a firm with several product lines with low substitutability (greater differences) is likely to having conducted more research (or to be short has more absorptive capacity) compared to a firm with product lines very close.

[2]Notice this incumbent innovation flow rate corresponds to an innovation intensity (innovation effort per product) $x_{ft} = \theta_f^{\gamma} \bar{q}_{ft}^{\gamma} (\frac{h_{ft}}{n_{ft}})^{1-\gamma}$. In fact, with the *assumption 2*, the same amount $\bar{q}_{ft}$ of absorptive capacity enters the per product innovation function instead of $\frac{\bar{q}_{ft}}{n_{ft}}$

With $\gamma \in (0,1)$, this function is linearly homogeneous in θ, $\bar{q}_{ft}$ and h_f and is concave in each of these argument separately. This means an incumbent which is as twice endowed with absorptive capacity as its counterpart of the same type, will need to hire twice as much skilled labour in order to innovate twice as fast.

If I drop out the subscript t, the above specification leads to the following R&D cost function

$$C(x_f, \theta_f, \bar{q}_f) = w^s n_f \bar{q}_f^{-\frac{\gamma}{1-\gamma}} x_f^{\frac{1}{1-\gamma}} \theta_f^{-\frac{\gamma}{1-\gamma}} \tag{3.8}$$

Proof: Appendix B2

For $\gamma \in (0,1)$, the R&D cost is a decreasing function of the type and the absorptive capacity of the firm.

Defining $\chi \equiv \frac{\gamma}{1-\gamma}$, and

$$G(x_f, \theta_f) \equiv x_f^{\frac{1}{1-\gamma}} \theta_f^{-\frac{\gamma}{1-\gamma}}$$

, the above R&D cost function for an incumbent firm can be written as follows:

$$C(x_f, \theta_f, \bar{q}_f) = w^s n_f \bar{q}_f^{-\chi} G(x_f, \theta_f) \tag{3.9}$$

The labour requirement for a per product innovation rate of x for an incumbent of type θ is then given by $\bar{q}_f^{-\chi} G(x, \theta)$, whereas in the basis model it is simply $G(x, \theta)$.

Given *the assumption 1*, successful innovation may be one of the following cases:

First, the firm‘s research team is successful in innovating upon an existing and active product line. Then, the innovation increases the productivity of that product line by λq_{jt}, then

$$q_{t+} = (1+\lambda) q_{jt}$$

$\lambda > 0$ is the innovation size parameter, measuring the proportional incremental improvement upon the existing technology and q_{t+} is the productivity of that product *j* after the innovation. The innovating firm then obtains the monopoly over the product line *j*, since its has the leading-edge technology.

Second, the firm‘s research team is successful in innovating over an inactive product line. This is the case for completely new product lines. The productivity of the new innovated product line is drawn from the stationary equilibrium productivity distribution $F(\hat{q})$

3.3.2 Innovation by entrants

At any time t, there are potential entrants conducting R&D activities in order to enter the market of a product line. When a potential entrant, in addition to ϕ units of skilled labour necessary for operations, hires h skilled labour into its research team, it successfully enters the market by innovating with the flow rate x^E given by:

$$x^{entry} = (\frac{\theta^E}{\bar{q}_t})^\gamma h^{1-\gamma} \tag{3.10}$$

With, $\theta^E \in \{\theta^H, \theta^L\}$ and

$$\bar{q}_t \equiv (\int_{N_t} q_j^{\epsilon-1} dj)^{\frac{1}{\epsilon-1}} \tag{3.11}$$

[3]

$\bar{q}_t$ is the average quality level of intermediate goods in the economy at time *t*.

From this flow rate of innovation, it can be noted that the economy‘s average productivity index acts as an extra cost, that potential entrants bear in order to enter the market. This expression suggests that by lessening the flow rate of innovation from potential entrants, higher productivity index of an industry withers the contribution of entry to the productivity growth in that industry, and then less reallocation of R&D resources.

This specification endows a potential entrant with the same flow rate of innovation as a *pseudo-incumbent* of *E-type*, but lacking capabilities that an average incumbent firm in the economy would have accumulated through its past R&D activities that helped that incumbent to operate product lines which average productivity index ($\bar{q}_{ft}$) is up to the economy‘s average productivity ($\bar{q}$).

The above innovation function specification leads to the following R&D cost function for a potential entrant.

$$C(x^E, \theta^E, \bar{q}) = w^s \bar{q}^\chi G(x^E, \theta^E) \tag{3.12}$$

with the function $G(x^E, \theta^E)$ being as defined above. Entrant‘s innovation cost is increasing in the economy‘s productivity index. The more an economy is advanced, the harder it will be for an outsider (potential entrant) to innovate within, or if we consider an industry, the more mature is an industry, the harder it is to enter it. The total skilled labour requirement for a potential entrant in order to achieve an innovation flow rate of x^E is then given by

$$h^{entry} = (\phi + \bar{q}^\chi G(x^E, \theta^E)) \tag{3.13}$$

which is different from the basis model.

Entrant can innovate upon existing product lines or be successful in more *radical innovation*(corresponding to inactive product lines). In any of the cases, the productivity of the product line on which the successful innovation takes on will be determined as specified above for incumbents.

[3]Similar innovation flow rate is used by Akcigit and Kerr (2015) for *external innovation* by incumbent firm, in a set up where they make a distinction between *external innovation* and *internal innovation*. This specification is as follows:$x = [\frac{R_x}{\chi \bar{q}}]^{\frac{1}{\psi}\mathbf{1}_{n>0}}$ where $1_{n>0}$ is an indicator function for firms having at least *1* product line, R_x is the cost of research, $\bar{q}$ the average quality level in the economy and with parameters $\chi > 0$ and $\psi > 1$

3.3.3 Firm and industry dynamics

Upon successful innovation, an entrant draws its type from $\theta \in \{\theta^H, \theta^L\}$. It is assumed that

$$Pr\left(\theta = \theta^H\right) = \alpha$$

and

$$Pr\left(\theta = \theta^L\right) = (1-\alpha)$$

with $\alpha \in (0,1)$ and $\theta^H > \theta^L > 0$. Then, any potential entrant bases its R&D policy upon the following free-entry condition:

$$max_{x^{entry} \geq 0}\left\{ -w^s\phi + x^{entry}\mathbb{E}V^{entry}\left(\hat{q},\theta\right) - w^s\bar{q}^{\chi}G\left(x^{entry},\theta^{entry}\right)\right\} = 0 \tag{3.14}$$

In stationary equilibrium, the measure of active potential entrant will be *m* leading to a total entry rate of $X^{entry} = mx^{entry}$

The firm's survival and then the reallocation of resources within industry and the whole industry dynamics are then driven by exogenous shocks and more importantly by an endogenous mechanism via productivity growth or new breakthrough from successful R&D activities. Regarding the endogenous mechanism, any product line can undergo obsolescence. At any point in time, given the unskilled labour wage rate, there is a threshold level of productivity for any product line to remain profitable. Any product line with productivity less than this threshold, conditional on firm's type becomes obsolete. The endogenous character of the obsolescence stems from the joint evolution of the unskilled wage rate and the productivity index of the economy as shown in (3.21). In other words, the endogenous obsolescence occurs when the productivity of a given product line lags behind to certain extend compared to the evolution of the unskilled wage rate within the economy. It should be noted that the fixed cost of operations as well determines the profitability of any product line, its obsolescence decision (exit) and then constitutes a point on which support policies may act on reallocation of resources in the model.

The creative destructive shock is endogenously shaped by entrants's innovation rate x^{entry} and incumbents' innovation rate whenever the innovation occurs in a product line not previously owned by the innovating incumbent. However, it should be noted that in this framework, the rate of this Schumpeterian force of creative destruction is assumed to be exogenous and of value τ.

Regarding the other exogenous shocks, it is assumed that high-type incumbents undergo transition shock at the exogenous flow rate ν and become of low-type. This is to capture the fact that some powerful innovative firms at some point in time become less innovative.

Additionally, there is possibility for any given firm's productivity to grow not necessarily via R&D activities, but by simply learning from production operations. It is then assumed that each active incumbent firm adds one more product line in its portfolio, taken from the inactive set of product line by receiving a positive productivity improvement shock at an exogenous rate ϱ per active product line.

Finally, each firm is subject to a negative destructive shock at the exogenous rate φ, and then exit the industry.

The above defined shocks can be summarized in the following table:

Shock	Rate
Negative destructive shock	φ
Transition from high-type to low-type shock	ν
Non-R&D productivity growth positive shock	ϱ
Creative destructive shock	τ

The rate of realization of each of the above mentioned shocks affects the value of product lines in the economy.

3.4 Value functions

Firm's value is in fact the ultimate objective variable that any choice either entry in new research line or production of new good have to maximize. It considers any source of certain or expected value added, of expenditure and factors source for discounting.

For a given firm in this economy, there are some exogenous shocks that may affect firm's product line and then its value, and must importantly there is an endogenous exit, which depends on the size and frequency of the firm's internal innovation rate relative to the growth rate of the economy's productivity index, depending on other firm's R&D outcomes. In other words, R&D policy decision of the firm takes into account the firm's value maximization objective, meanwhile determining the growth rate of the economy via the productivity increase at the firm-level.

Since I am interested in analysing the model in the stationary equilibrium, all the growing variables are normalized by (Q). Then the normalized value of a generic value X becomes $\tilde{X} = \frac{X}{Q}$.

As exogenous shocks affecting the firm's value function, we have the productivity growth shock not related to R&D activities which rate is ϱ, the negative destructive shock which rate is φ and the transition from high-type shock that concerns only high-type firm, which rate is ν. The firm's own innovation shock, which rate is x endogenously affects its value as well and the creative destructive shock that fundamentally is endogenous, but assumed here to be of the exogenous rate τ, resulting from other firms' outcomes in research.

Then, the stationary equilibrium values for low-type and high-type are respectively given by:

$$\begin{aligned} r\tilde{V}_l\left(\hat{Q}\right) = max\Bigg\{0, max_x\Bigg[\tilde{\pi}\left(\hat{q}\right) - w^s\phi + \\ \sum_{\hat{q}\in\hat{Q}} \left[\tau[\tilde{V}_l\left(\hat{Q}/\hat{q}\right) - \tilde{V}_l\left(\hat{Q}\right)\right] + \frac{\partial\tilde{V}_l\left(\hat{Q}\right)}{\partial\hat{q}}\frac{\partial\hat{q}}{\partial w^u}\frac{\partial w^u}{\partial t}] \\ - n\tilde{w}\bar{q}_f^{-\chi}G\left(x,\theta^L\right) + nx[\mathbb{E}\tilde{V}_l\left(\hat{Q}\cup\hat{q}(1+\lambda)\right) - \tilde{V}_l\left(\hat{Q}\right)] \\ + n\varrho[\mathbb{E}\tilde{V}_l\left(\hat{Q}\right) - \tilde{V}_l\left(\hat{Q}\right)] \\ + \varphi[\mathbb{E}\tilde{V}_l\left(0 - \tilde{V}_l\left(\hat{Q}\right)\right)]\Bigg]\Bigg\} \end{aligned} \tag{3.15}$$

and

$$r\tilde{V}_h\left(\hat{Q}\right) = max\Bigg\{0, max_x\Bigg[\tilde{\pi}\left(\hat{q}\right) - w^s\phi +$$
$$\sum_{\hat{q}\in\hat{Q}}\left[\tau[\tilde{V}_h\left(\hat{Q}/\hat{q}\right) - \tilde{V}_h\left(\hat{Q}\right)] + \frac{\partial\tilde{V}_h\left(\hat{Q}\right)}{\partial\hat{q}}\frac{\partial\hat{q}}{\partial w^u}\frac{\partial w^u}{\partial t}\right]$$
$$- n\tilde{w}\bar{q}_f^{-\chi}G\left(x,\theta^H\right) + nx[\mathbb{E}\tilde{V}_h\left(\hat{Q}\cup\hat{q}(1+\lambda)\right) - \tilde{V}_h\left(\hat{Q}\right)]$$
$$+ n\varrho[\mathbb{E}\tilde{V}_h\left(\hat{Q}\right) - \tilde{V}_h\left(\hat{Q}\right)]$$
$$+ \varphi[\mathbb{E}\tilde{V}_h\left(0 - \tilde{V}_h\left(\hat{Q}\right)\right)] + \nu[\mathbb{I}_{\hat{Q}>\hat{Q}_{l,min}}.\tilde{V}_l\left(\hat{Q}\right) - \tilde{V}_h\left(\hat{Q}\right)]\Bigg]\Bigg\} \tag{3.16}$$

where $\hat{Q}\cup\{\hat{q}_{j'}\}$ is the new portfolio of the firm after successfully innovating in the product line j'. Similarly,$\hat{Q}/\{\hat{q}_j\}$ denotes a loss of product line with technology $\hat{q}_j$ from firm *f's* portfolio $\hat{Q}$ due to creative destruction.

These value functions for both types are very similar, except their last line. Their interpretation is as follows:
The left-hand side gives the flow value of a *k*-type firm with a set of product lines given by $\hat{Q}$ discounted at the rate *r*, and the right-hand side equals the components that make up this flow value. For the right-hand side, the first line is the net operating profit. The first element (with sum symbol) of the second line gives the change in k-type firm's value attributed to the creative destruction at the rate τ. The second and last element of that second line, that is $\frac{\partial\tilde{V}_k(\hat{Q})}{\partial\hat{q}}\frac{\partial\hat{q}}{\partial w^u}\frac{\partial w^u}{\partial t}$, expresses the variation in *k*-type firm's value due to the variation in a product line relative productivity following variation in time of the unskilled wage rate in the economy. The first element of the third line considers the cost or research. The second element of the third line expresses the variation in *k*-type firm's value when it succeeds in increasing a product line productivity from $\hat{q}$ to $\hat{q}(1+\lambda)$ at the rate *x* per product line. The fourth line is the change in *k*-type firm's value due to an exogenous gain of product line for reasons not related to research activities, at the rate ϱ. The first element of the last line gives the variation in k-type firm's value when it exits after been hit by a negative shock at the rate φ. The last element of the last line, which concerns only the high type firm, is the variation in high-type firm's value when hit by the transition shock at the rate ν

Multi-product firms receive profit from each of their product line and the obsolescence decisions for product lines are independent. At the same time, since research is undirected (*assumption 1*), product line quality improvement is equally likely for each of the product line. This suggests the additive form of value function for a k-type firm as states by the following Lemma 1:

Lemma 1: *The value function of a* $k\in\{h,l\}$ *type firm takes an additive form*

$$\tilde{V}_k\left(\hat{Q}\right) = \sum_{\hat{q}\in\mathbf{q}}\Upsilon^k\left(\hat{q}\right)$$

where $\Upsilon^k(\hat{q})$ is the franchise value of a product line with relative productivity $\hat{q}$ to a firm of type k. Moreover, $\Upsilon^k(\hat{q})$ is strictly increasing and firms follow a cut-off rule for their obsolescence decision such that

$$\iota^k(\hat{q}) = \begin{cases} = 1 & \text{if } \hat{q} > \hat{q}_{k,min} \\ = 0 & \text{if } \hat{q} < \hat{q}_{k,min} \\ \in [0,1] & \text{Otherwise} \end{cases}$$

The above obsolescence cut-off rule is the central margin for the endogenous exit for less productive firms. As long as the relative productivity of a given product line is above the threshold conditional on firm-type, it remains active. Once under the threshold, the product becomes obsolete and if it was the only product for the firm, the firm exits the economy. Then any subsidy to incumbent firm is going to affect the threshold point at which the firm must exit. Precisely, any subsidy would endow a product line with a lower threshold productivity level than otherwise, and would postpone by then the product line obsolescence.

With *Lemma 1*, I can now obtain the value function of each *k*-type firm via the franchise value of each of its operating product line, this in terms of differential equations as states in the following Lemma 2:

Lemma 2: the franchise values of owning a product line of relative productivity $\hat{q}$ by low-type firm and high-type firm respectively are given by the following differential equations:

$$(r+\tau+\varphi)\Upsilon_l(\hat{q}) - \frac{\partial \Upsilon^l(\hat{q})}{\partial \hat{q}}\frac{\partial \hat{q}}{\partial w^u}\frac{\partial w^u}{\partial t} = \left\{\Pi \hat{q}^{\epsilon-1} - \tilde{w}^s\phi + \omega^l\right\} \text{if } \hat{q} > \hat{q}_{l,min} \tag{3.17}$$
$$\Upsilon_l(\hat{q}) = 0, \text{Otherwise}$$

and

$$(r+\tau+\varphi)\Upsilon_h(\hat{q}) - \frac{\partial \Upsilon^h(\hat{q})}{\partial \hat{q}}\frac{\partial \hat{q}}{\partial w^u}\frac{\partial w^u}{\partial t} = \left\{\Pi \hat{q}^{\epsilon-1} - \tilde{w}^s\phi + \Omega^h + \nu[\mathbb{I}_{\hat{q}>\hat{q}_{l,min}}.\upsilon^l(\hat{q}) - \upsilon^h(\hat{q})]\right\} \text{if } \hat{q} > \hat{q}_{h,min} \tag{3.18}$$
$$\Upsilon_h(\hat{q}) = 0, \text{Otherwise}$$

where Π and Ω^k are respectively defined as follows:

$$\Pi \equiv \frac{1}{\epsilon-1}[\frac{\epsilon-1}{\epsilon}]^\epsilon$$

$$\Omega^k \equiv max_x\left\{ -\tilde{w}^s\bar{q}_f^{-\chi}G\left(x_f,\theta^k\right) + x_f\mathbb{E}\Upsilon^k\left(\hat{q}(1+\lambda)\right) + \varrho\mathbb{E}\Upsilon^k\left(\hat{q}\right)\right\} \text{for } k \in \{L,H\}$$

The option value for a *k*-type firm is the expected net profit from engaging in research activities in a given product line, either an existing one or in an exploratory field. It follows from the k-type free-entry condition stated further above.

These equations state the balance sheet value of a product line as equal to its franchise value net of the total effective discount factor minus its variation due to the variation in time of the unskilled labour wage rate in the economy, as long as the relative productivity is above the obsolescence threshold.

From (3.21), we can note that the unskilled wage rate varies proportionately with the economy‘s productivity index. This variation in unskilled wage rate is then the underpinning of the endogenous obsolescence of product lines.

3.5 Equilibrium

The rational behaviour of different agents, namely households and firms at a given time t, leads to specific values for the different variables, the equilibrium of the model. I am not interested in the time path of these equilibrium variables, but at their value at a given point in time, and how these variables in general and the growth rate in particular would react to some policy shocks.

The household's intertemporal choice between consumption and asset accumulation and their remuneration delivers the following standard Euler equation:

$$\frac{\dot{C}}{C} = \frac{r-\rho}{\vartheta} \tag{3.19}$$

Proof: Appendix A1

The implicit final good market is competitive and each intermediate good price is given by its marginal product from (3.2), which constitutes the inverse demand facing the monopolist producing it. This inverse demand for a product j is given by

$$p_j = C^{\frac{1}{\epsilon}} c_j^{-\frac{1}{\epsilon}}, \forall j \in [0,1]$$

Innovation is assumed to be drastic, therefore, for a product line in which the monopolist firm has the leading-edge technology, its objective is to solve the following maximization problem:

$$\pi\left(q_j\right) = max_{c_j}\{\left(C^{\frac{1}{\epsilon}} c_j^{-\frac{1}{\epsilon}} - \hat{q}_j^{-1}\right) c_j\}$$

This maximization problem determines the optimal quantity to be produced by the monopolist and therefore the corresponding monopolistic price as follows:

$$p_j = \frac{\epsilon}{\left(\epsilon-1\right)\hat{q}_j} and c_j = [\frac{\epsilon-1}{\epsilon}\hat{q}_j]^{\epsilon} \tag{3.20}$$

Proof: Appendix A2

The corresponding equilibrium profit given by the optimal price and quantity is:

$$\pi\left(\hat{q}_j\right) = \frac{\hat{q}_j^{\epsilon-1}}{\epsilon-1}[\frac{\epsilon-1}{\epsilon}]^{\epsilon} C$$

The profit from a product line (intermediate good) is increasing in its productivity and the aggregate final good demand.

Finally, the stationary equilibrium unskilled wage rate in production is determined from (3.2), using the results of (3.20).

$$w_u = \frac{\epsilon - 1}{\epsilon} Q \tag{3.21}$$

where Q is given in (3.5)

Proof: Appendix B1

This expression suggests that the unskilled wage rate grows at the same rate as the economy's productivity index. Then, product line that the productivity fails to be upgraded will be more likely to become obsolete with the growth of the aggregate productivity level in the economy.

The research decision of any incumbent conditional on its type is given by

$$x_k = \theta^k \bar{q}_f [\frac{(1-\gamma)\mathbb{E}\Upsilon^k(\hat{q}(1+\lambda))}{\tilde{w}^s}]^{\frac{1-\gamma}{\gamma}} \textit{for } k \in \{L, H\} \tag{3.22}$$

Proof: Appendix C1

The R&D effort of incumbent firm is positively affected by its (proxy of) absorptive capacity, being in line with Nieto and Quevedo (2005).

At the stationary equilibrium, the unskilled wage rate will determine the threshold of productivity level at which each type of product line can profitably remain active. Under this threshold, the product line becomes obsolete and the firm loses its full franchise value. This threshold value $\hat{q}_{k,min}$, conditional on firm type is given by:

$$\hat{q}_{k,min} = \left(\frac{\tilde{w}^s \phi - \Omega^k}{\Pi} \right)^{\frac{1}{\epsilon - 1}} \textit{for } k \in \{L, H\} \tag{3.23}$$

Proof: Appendix C2

Given the above R&D policy function, the solution to the differential equations of **Lemma 2** is provided by the following proposition.

Proposition 1: *Let g and $\tilde{w}^s$ be the stationary equilibrium growth rate of the economy and the normalized skilled wage rate respectively. Moreover, given the following definition,*

$$F_k(x) \equiv [1 - \left(\frac{\hat{q}_{k,min}}{\hat{q}} \right)^x]$$

the franchise value of a product line with relative productivity $\hat{q}$ for a low-type firm is given by:

$$\Upsilon_l(\hat{q}) = \frac{\Pi \hat{q}^{\epsilon-1}}{\Psi + (\epsilon - 1) g} F_l \left(\frac{\Psi + (\epsilon - 1) g}{g} \right) + \frac{\Omega^l - \tilde{w}^s \phi}{\Psi} F^l \left(\frac{\Psi}{g} \right)$$

, where $\Psi \equiv (r + \tau + \varphi)$.

Proof: Appendix D

Similarly, the franchise value of a product line with relative productivity $\hat{q}$ for a high-type firm is given by:

$$\Upsilon_h(\hat{q}) = \begin{cases} = \frac{\Pi \hat{q}^{\epsilon-1}}{\Psi+\nu+(\epsilon-1)g} F_h\left(\frac{\Psi+\nu+(\epsilon-1)g}{g}\right) - \frac{\tilde{w}^s\phi - \Omega^h}{\Psi+\nu} F_h\left(\frac{\Psi+\nu}{g}\right) & \textit{for } \hat{q} \in [\hat{q}_{h,min}, \hat{q}_{l,min}] \\ = \left\{ \frac{\Pi \hat{q}^{\epsilon-1}}{\Psi+\nu+(\epsilon-1)g} F_h\left(\frac{\Psi+\nu+(\epsilon-1)g}{g}\right) + \frac{\Omega^h - \tilde{w}^s\phi}{\Psi+\nu} F^h\left(\frac{\Psi+\nu}{g}\right) \right. & \\ + \frac{\Pi \hat{q}^{\epsilon-1}}{\Psi+(\epsilon-1)g} F_l\left(\frac{\Psi+(\epsilon-1)g}{g}\right) + \frac{\Omega^l - \tilde{w}^s\phi}{\Psi} F^l\left(\frac{\Psi}{g}\right) & \\ \left. - \frac{\Pi \hat{q}^{\epsilon-1}}{\Psi+\nu+(\epsilon-1)g} F_l\left(\frac{\Psi+\nu+(\epsilon-1)g}{g}\right) + \frac{\Omega^h - \tilde{w}^s\phi}{\Psi+\nu} F_l\left(\frac{\Psi+\nu}{g}\right) \right\} & \textit{for } \hat{q} \geq \hat{q}_{l,min} \end{cases}$$

Proof: Appendix E

3.6 Labour market and stationary equilibrium

The total supply of unskilled labour is normalized at *1*, which equals the total demand in the equilibrium. Then the unskilled labour market clearing condition is

$$\int_N \iota\left(\hat{q}_j\right) dj[\frac{\epsilon-1}{\epsilon}\frac{1}{w}]^{\epsilon} C \int_N q_j^{\epsilon-1} dj = 1 \tag{3.24}$$

Regarding the skilled labour market, the total demand is given by entrants and incumbents' needs in skilled labour, both for research and fixed costs of operations. This market is then conditioned by the type shares of product lines and the related productivity distributions.

Then, let denote by Φ^h, Φ^l and Φ^{np} the shares of product lines belonging to high-type, low-type and inactive product lines respectively, with $\Phi^h+\Phi^l+\Phi^{np}=1$. Given the stationary equilibrium productivity distributions of any k-type firm $F_k\left(\hat{q}\right)$ on $[\hat{q}_{k,min},\infty)$, the skilled labour market clearing condition is given by:

$$m[\phi + G\left(x^{entry},\theta^{entry}\right)] + \int_N \left(\sum_{k\in\{k,l\}} \Phi^k[h_k(w^s)+\phi]\right) = L^S \tag{3.25}$$

Finally, using (3.5), (3.20) and (3.21), the previous labour market condition gives

$$Y = C = Q \tag{3.26}$$

The exogenous and endogenous shocks mentioned above shape the reallocation of resources, of market shares and finally the whole industry dynamics. The productivity distributions conditional on firm type evolve whenever any of these shocks comes to be realized. The amount of skilled workers hired being dependent on the firm's type, there is need to characterize the measure of the share of product lines owned by each type of firm in equilibrium. This is done by the following equations:

$$\begin{aligned}\left(\alpha X^{entry}+\Phi^h x^h\right)\left(1-\Phi^h\right)+\varrho\Phi^h = \Big\{\Phi^h\left(\nu+\phi+X^{entry}\left(1-\alpha\right)+\Phi^l x^l\right) \\ +\Phi^h \hat{q}_{h,min} g f_h\left(\hat{q}_{h,min}\right)\Big\}\end{aligned} \tag{3.27}$$

$$\begin{aligned}&\Big\{\left(X^{entry}(1-\alpha)+\Phi^l x^l\right)\left(1-\Phi^l\right) \\ &+\Phi^h\nu[1-F_h(\hat{q}_{l,min})]+\nu\Phi^l\Big\} = \Phi^l\left(\phi+X^{entry}\alpha+\Phi^h x^h\right)+\Phi^l\hat{q}_{l,min} g f_l\left(\hat{q}_{l,min}\right)\end{aligned} \tag{3.28}$$

$$\Bigg\{ \phi(1-\Phi^{np}) + \Phi^h \hat{q}_{h,min} g f_h\left(\hat{q}_{h,min}\right)$$

$$+ \Phi^l \hat{q}_{l,min} g f_l\left(\hat{q}_{l,min}\right) + \Phi^h \nu F_h(\hat{q}_{l,min}) \Bigg\} = \varrho(1-\Phi^{np}) + \Phi^{np}\left(X^{entry} + \Phi^h x^h + \Phi^l x^l\right) \tag{3.29}$$

In each equation, the left-hand side expresses the inflows into the product lines of type *h*, *l* and *np* (which are respectively controlled by high-type, low-type firms and inactive) and the right-hand side expresses the outflows. The main terms making up these equations can be interpreted as follows, which interpretations provide intuition for the remaining terms:

High-type

αX^{entry}: High-type entrants innovation upon an inactive product line or previously belonging to a low-type (total share of such product lines is $\left(1-\Phi^h\right)$.

$\Phi^h x^h$: high-type incumbents innovation upon a product line not previously belonging to a high-type.

$\varrho\Phi^h$: High-type innovation not driven by R&D

$\Phi^h x^h + \Phi^l \hat{q}_{l,min} g f_l\left(\hat{q}_{l,min}\right)$: Outflow (due to obsolescence) of a product line controlled so far by high-type when relative productivity $\hat{q} < \hat{q}_{h,min}$, driven by the increase in the economy's wide productivity index *g*.

$\Phi^h X^{entry}(1-\alpha)$: Product lines taken away from high-type by a low-type entrant.

$\Phi^h \Phi^l x^l$: Product lines taken away from high-type by a low-type incumbent.

$\Phi^h \nu$: High-type product lines transitioned to low-type.

$\Phi^h \varphi$: High-type product lines destroyed by the negative exogenous destructive shock.

Low-type

$\Phi^h \nu[1 - F_h(\hat{q}_{l,min})]$: Transition of product lines from high-type to low-type when hit by the negative shock with rate ν provided that $\hat{q}_h < \hat{q}_{l,min}$

The remaining specification for solving the market clearing condition is to characterize the stationary equilibrium productivity distributions conditional on firm type. These distributions at the stationary equilibrium are in such away that the flow into and out of any interval of productivity are equalized. This is done by the following lemma.

Lemma 3: *The stationary equilibrium (invariant) productivity distributions of active product lines of low-type and high-type satisfy the following equations*:

$$
g\hat{q}f_l(\hat{q}) = \begin{cases} g\hat{q}_{h,min}f_h(\hat{q}_{h,min}) + (\tau+\phi+\nu)[F_h(\hat{q}) - F_h(\hat{q}_{h,min})] \\ [\ -\left(\dfrac{\Phi^h x^h + X^{entry}\alpha\text{ffl}}{\Phi^h}\right)\left(\begin{matrix}\Phi^h F_h(\dfrac{\hat{q}}{1-\lambda}) + \Phi^l F_l(\dfrac{\hat{q}}{1-\lambda}) \\ +(1-\Phi)F(\hat{q})\end{matrix}\right)]\textit{for } \hat{q}_{l,crit} < \hat{q} \\ g\hat{q}_{h,min}f_h(\hat{q}_{h,min}) + (\tau+\phi+\nu)[F_h(\hat{q}) - F_h(\hat{q}_{h,min})]\textit{for } \hat{q}_{h,min} < \hat{q} \leq \hat{q}_{h,crit} \end{cases} \tag{3.30}
$$

and

$$
g\hat{q}f_h(\hat{q}) = \begin{cases} = g\hat{q}_{h,min}f_h(\hat{q}_{h,min}) + (\tau+\nu+\rho))[F_h(\hat{q}) - F_h(\hat{q}_{h,min})] \\ -\frac{(\Phi x^h + \alpha X^{entry})}{\Phi^h}[\Phi^h F_h(\frac{\hat{q}}{1+\lambda}) + \Phi^l F_l(\frac{\hat{q}}{1+\lambda}) + \Phi^{np}F(\hat{q})]\textit{for } \hat{q} > \hat{q}_{h,min}(1+\lambda) \\ = g\hat{q}_{h,min}f_h(\hat{q}_{h,min}) + (\tau+\nu+\rho)[F_h(\hat{q}) - F_h(\hat{q}_{h,min})] \\ \textit{for } \hat{q}_{h,min}(1+\lambda) < \hat{q} \leq \hat{q}_{h,min}(1+g\Delta t) \end{cases} \tag{3.31}
$$

Where $\Phi \equiv \Phi^h + \Phi^l$ is the measure of active product lines and $\hat{q}_{l,crit} \equiv \hat{q}_{l,min}(1+\lambda)$

Proof: Appendix F

3.7 Aggregate growth

One of the aims of the above described framework is to link the aggregate growth rate of the economy to the productivity growth at the firm-level. In this economy as shown by (3.26), the output growth is the consequence of intermediate goods productivity improvement and discovery of new ones, each intermediate good contributing in the way expressed by (3.5). Moreover, given the stationary equilibrium productivity distributions conditional on firm type, the productivity index of product lines owned by high-type firms and the productivity index of product lines owned by low-type firms grow at the same rate.

If I denote the type-specific productivity indices by

$$\tilde{Q}_t^k = \int_{N_t^s} q_{jt}^{\epsilon-1} dj$$

where $k \in \{h, l\}$, we have the firm-type productivity index growth given by

$$g^h = \frac{\frac{\partial \tilde{Q}_t^h}{\partial t}}{\tilde{Q}_t^h}$$

and the equality holds for the aggregate output growth rate.

$$g = \frac{g^h}{\epsilon - 1} = \frac{g^l}{\epsilon - 1}$$

The next proposition gives the growth rate of the economy stemming from the firm-level.

Proposition 2: *The growth rate of the economy is then equal to*

$$g = \frac{\left(x^h + \alpha X^{entry}\right)\left[(1+\lambda)^{\epsilon-1}(1+\frac{1}{\Gamma}) + \kappa_h\right] + \varrho \Phi^h \kappa_h - (\tau + \nu + \phi)}{\epsilon - 1 + \hat{q}_{h,min} f_h(\hat{q}_{h,min})} \tag{3.32}$$

Where $\kappa_k \equiv \frac{\Phi^{np} \mathbb{E}_{F q_t^{\epsilon-1}}}{\tilde{Q}_t^k}$

is the ratio of the productivity index of inactive product lines to *k*-type productivity index. The ratio of productivity index of low-type to that of high-type active product lines $\Gamma \equiv \frac{\tilde{Q}^h}{\tilde{Q}^l}$ is solution to

$$\left\{ \begin{aligned} &(x^h + \alpha X^{entry})[(1+\lambda)^{\epsilon-1}(1+\frac{1}{\Gamma}) + \kappa_h] \\ &\qquad + \varrho \kappa_h [\Phi^h - \Phi^l \Gamma] \end{aligned} \right\} = \left\{ \begin{aligned} &\left(x^l + (1-\alpha) X^{entry}\right)\left[(1+\lambda)^{\epsilon-1}(1+\frac{1}{\Gamma}) + \kappa_l\right] \\ &\qquad + \nu[1 + [1 - F_h(\hat{q}_{l,min})]\Gamma] \\ &\qquad + g[\hat{q}_{h,min} f_h(\hat{q}_{h,min}) - \hat{q}_{l,min} f_l(\hat{q}_{l,min})] \end{aligned} \right\}$$

This equality follows the fact that in the equilibrium, the productivity index of product lines owned by high-type firms grows at the same rate as the productivity index of product lines owned by low-type firms.

Proof: Appendix G

The above aggregate growth rate is obtained from high-type product lines and is exactly equivalent to the one derived from the low-type product lines. The numerator of this growth rate expression comprises the contribution of incumbents and entrants via successful innovation in the productivity distribution, the positive impact of productivity growth for non-R&D related activities net of negative shocks that these product lines are subject to. The denominator adjusts for the improvement of productivity due to obsolescence. The equilibrium values of x^h and X^{entry} are likely to be different from what are obtained in Acemoglu et al. (2013), conditional on other parameters of the model.

Definition: Stationary equilibrium A stationary equilibrium of this economy is a list of the following objects

$\{y_j, p_j, l_j, \tilde{V}_l, \tilde{V}_h, \hat{q}_{h,min}, \hat{q}_{l,min}, x^h, x^l, x^{entry}, h^h, h^l, h^{entry}, m, \Phi^h, \Phi^l, \Phi^{np}, F_h(\hat{q}), F_l(\hat{q}),$ $w^s, w^u, g, r, \bar{q}\}$

such that [i] y_j and p_j maximizes profits as in (3.20) and the labour demand l_j satisfies (3.3);[ii] $\tilde{V}_h$ and $\tilde{V}_h$ are given by the low-type value function in (3.15)and (3.16); [iii] $(\hat{q}_{h,min}, \hat{q}_{l,min})$ satisfy the cut-off rule in (3.23); [iv]x^h and x^l are given by the the R&D policy functions in (3.22) and x^{entry} and m satisfy the free-entry condition in (3.14); [v] the skilled worker demands h^h, h^l and h^{entry} satisfy (3.6) and (3.13); [vi] The product line shares $(\Phi^h, \Phi^l, \Phi^{np})$ satisfy ((3.27)) - (3.29); [vii] The stationary equilibrium productivity distributions $(F_h(\hat{q}), F_l(\hat{q}))$ satisfy (3.30) and (3.31); [viii] The growth rate is given by (3.32); [ix] The interest rate satisfies the Euler equation (3.32); [x] w^s and w^u are consistent with labour market clearing for unskilled and skilled workers as given by (3.24) and (3.25) and [xi] $\bar{q}$ is given by (3.11); [xii] $\bar{q}_f$ given by (3.7).

3.8 Discussion

At this stage of my analysis, I stand on the likely results my framework may yield in light with results obtained by the basis model and the pointed mechanisms.

Acemoglu et al. (2013) conducts some policy experiments with the estimated parameters of their model.

First, subsidies to incumbents' operation costs reduce growth: subsidies keep low-type incumbents in activities more than a social planner would, by lowering their exit via obsolescence; by then, they exacerbate skilled wage rate and discourage the more efficient entrants.

Second, subsidies to incumbents R&D reduces growth: greater R&D for incumbents due to subsidies not only discourages entry by increasing creative destruction, but increases the skilled wage rate by greater demand of skilled labour. Resources are then reallocate from entrants to incumbents (perversely to low-type incumbents).

Third, subsidies to entry have positive effects (of trivial size) on growth: the higher demand for skilled labour induced by entry increases their wage rate and increases the threshold of obsolescence as well, especially for low-type incumbents. There is then a positive selection effect, reallocating resources from less efficient incumbents to more efficient entrants. Their proposed optimal policy is then a fairly small subsidy to incumbents' R&D coupled with a large tax rate (approximately 26% of their revenues) on incumbents' operations will yield a result close to the social planner's result. Low-type incumbents will then be forced to exit and free up then the R&D resources that will be reallocated to more efficient entrants.

As it as been shown in *section 3* above, with the specification taking into account the concept of firm's absorptive capacity, in equilibrium, the labour requirement for a per product innovation rate for a given incumbent is more likely to be lower compared to the reference model. In my model, there is a new comparative advantage in innovating recognized to incumbents, and importantly including low-type incumbents which are at the source of the strong negative selection effect induced by R&D subsidies in Acemoglu et al. (2013)'s model. I expect the set up framework to produce different results from Acemoglu et al. (2013) through the following channels:

First, in equilibrium, a given low-type incumbent needs less skilled workers for the same innovation rate; Indeed, in equilibrium, the gap in research efficiency between low-type incumbent and potential entrant is likely to be narrowed. In fact, on the one hand, a low-type incumbent benefits now from its absorptive capacity in innovating and on the other hand, potential entrants are "penalized" by their lack of absorptive capacity. Then, although there is still a negative selection effect, it is expected not to be as strong as in the basis model since here, low-type are more likely to be more efficient and entry more difficult. In other words, if any, the lost incurred by the economy due to subsidies to incumbents' R&D, keeping then for long time resources (skilled labour) within an inefficient use by low-type incumbents instead of reallocating them to entrant is expected not be be has much as in the basis model.

Second, overall incumbent demand (Or high-type skilled labour requirement for a per product innovation rate) in skilled workers is lower; In fact, incumbent R&D subsidies will increase their R&D. But unlike in the basis model, in order to achieve the same innovation rate, they will need lesser skilled labour in equilibrium as shown by the skilled labour requirement for a per product innovation rate of x given by $\bar{q}^{-\chi}G(x,\theta)$ compared to $G(x,\theta)$ in the basis model, subject to a constant trend for $G(x,\theta)$ in the worst of cases compared to the basis model. This will then be less upwards pressure on the skilled labour wage rate that was pointed out by Acemoglu et al. (2013) as responsible for the reduction of R&D by both entrants and incumbents.

Third, entrants' profitability is lessening by the average level of economy's productivity, increasing then the innovation incentives for incumbents. In fact, another channel of results improvement may come from the fact that less innovation rate for potential entrants will reduce the overall creative destruction from incumbent's view and then discourage less innovation by incumbents either without or with the subsidies to research.

In sum, my framework via the above mentioned channels is then more likely to yield a decomposition of the economy's growth rate with greater contribution from incumbents compared to entrants in contrary to Acemoglu et al. (2013), but in accords with Akcigit and Kerr (2015), Acemoglu and Cao (2015) and Garcia-Marcia, Hsieh, and Klenow (2015).

Chapter 4

Conclusion

I have set up a theoretical framework that may shed more light on the effects of R&D subsidies on the aggregate growth rate of economy, by including the concept of firm's absorptive capacity into a baseline model by Acemoglu et al. (2013), following Klette and Kortum (2004). Considering this concept of *absorptive capacity* in Cohen and Levinthal (1989)'s spirit led to defining a proxy of it taking into account the number of product lines firm operates, their quality level and their closeness. I have motivated why this measure is different from the concept of *knowledge capital* used by Klette and Kortum (2004) and Acemoglu et al. (2013) and why it may capture in a more accurate way the firm's current capability in innovation efficiency gained from past research activities. Different innovations functions for incumbents and potential entrants have being specified in light to the concept. By including the concept of absorptive capacity into the framework, I introduced an endogenous time variation in firm's innovation capacity in addition to the exogenous variation induced by the transition of high-type firms to low-type.

The obtained framework features heterogeneity in size and innovation capabilities and allows for endogenous entry and exit. A product line which productivity growth lags behind comparatively to the growth rate of the unskilled labour wage in the economy is more likely to become obsolete, and in absence of any support measure like subsidies, the corresponding firm is more likely to exit the economy. The aggregate output growth stems from the growth of individual firm's intermediate goods productivity. Unlike the reference model, in light with the notion of absorptive capacity and supported by recent empirical findings by GarciaMarcia, Hsieh, and Klenow (2015) and Akcigit and Kerr (2015), an incumbent has clear advantage over a potential entrant in innovation likelihood, other innovation determinants held constant. Therefore, although the obtained growth rate has the same structure as in Acemoglu et al. (2013), the content of incumbents' rate of innovation and entrants' rate of entry is different. This is more likely to deliver different empirical results from the reference model.

The main future direction following this study would be to confront this model to the same data used in the reference model. A second line may be to extend the model to study the within industry reallocation, the survival dynamics at different stages of industrial development. Finally, the model may be extended to include the transition of non-innovative firms to innovative ones and shed then more light on policies that may be beneficial for a take-off of a vibrant and innovative manufacturing sector in backwards economies.

Appendix A

Euler equation, equilibrium price and profits

Appendix A1: Euler Equation

A representative household would chose a consumption value that maximizes (3.1) subject to its wealth accumulation constraint. The consumption price is normalized at 1. I drop out the subscript t and the resulting Hamiltonian is

$$H = U_0 + \lambda \dot{W} \tag{A.1}$$

$$= \frac{C^{1-\nu} - 1}{1 - \nu} + \lambda \left([w^u + w^s L^S] + Wr - C \right) \tag{A.2}$$

With consumption as control variable and wealth as state variable, we obtain the following relations for solving the maximization problem by the Maximum principle Method:

$$\begin{cases} H = U_0 + \lambda \dot{W} = \frac{C^{1-\nu}-1}{1-\nu} + \lambda(Y + Wr - C) \\ \frac{\partial \lambda}{\partial t} = \rho\lambda - \frac{\partial H}{\partial W} and \lambda(T).W(T)exp^{-\rho T} = 0 \\ \frac{\partial H}{\partial C} = 0 \end{cases} \tag{A.3}$$

$$\frac{\partial H}{\partial C} = C^{-\nu} - \lambda$$

$$\Rightarrow C = (\lambda)^{-\frac{1}{\nu}}$$

$$\frac{\partial \lambda}{\partial t} = \rho\lambda - r\lambda = (\rho - r)\lambda$$

$$
\begin{aligned}
\ln C &= -\frac{1}{\nu}\ln\lambda \\
\frac{\partial \ln C}{\partial C} &= \frac{\dot{C}}{C} = \frac{-1}{\nu}\frac{\dot{\lambda}}{\lambda} \\
\Rightarrow \frac{\dot{C}}{C} &= \frac{-1}{\nu}\frac{(\rho - r)\lambda}{\lambda} \\
&= -[\frac{\rho - r}{\nu}] \\
\Rightarrow \frac{\dot{C}}{C} &= \frac{r-\rho}{\nu}
\end{aligned}
$$

which gives the resulting standard Euler Equation.

Appendix A2: Intermediate good price and monopolist profits.

From the final good producer side, the price of each intermediate good she uses is its marginal product, since the final good (product) market is competitive, each factor is remunerated at its marginal product. This price will correspond to the inverse demand facing the monopolist producing the given intermediate good. In turn, the monopolist will set a quantity that maximizes its profit and thus, there will be a corresponding equilibrium price for any given intermediate good.

$$
\begin{aligned}
p_j &= \frac{\partial C(t)}{\partial c_j(t)} \\
&= \frac{\epsilon}{\epsilon-1}\frac{\epsilon-1}{\epsilon} c_j^{\frac{\epsilon-1}{\epsilon}-1}\left(\int_{N(t)} c_j^{\frac{\epsilon-1}{\epsilon}}\, dj\right)^{\frac{\epsilon}{\epsilon-1}-1} \\
&= c_j^{-\frac{1}{\epsilon}}\left(\int_{N(t)} c_j^{\frac{\epsilon-1}{\epsilon}}\, dj\right)^{\frac{1}{\epsilon-1}\frac{\epsilon}{\epsilon}} \\
&= c_j^{-\frac{1}{\epsilon}}\left(\int_{N(t)} c_j^{\frac{\epsilon-1}{\epsilon}}\, dj\right)^{\frac{\epsilon}{\epsilon-1}\frac{1}{\epsilon}} \\
&= c_j^{-\frac{1}{\epsilon}} C^{\frac{1}{\epsilon}}
\end{aligned}
$$

$\forall j \in [0,1]$

The monopolist producer would determine its optimal quantity to be produced from the following profit maximization problem:

$$
max_{c_j}\{[c_j^{-\frac{1}{\epsilon}}C^{\frac{1}{\epsilon}} - \frac{w^u}{q_j}]c_j\} = max_{c_j}\{[c_j^{-\frac{1}{\epsilon}}C^{\frac{1}{\epsilon}} - \hat{q}_j^{-1}]c_j\}
$$

$$
\frac{\partial \Pi_j}{\partial c_j} = \frac{\epsilon-1}{\epsilon}C^{\frac{1}{\epsilon}}c_j^{-\frac{1}{\epsilon}} - \hat{q}_j^{-1} = 0
$$

$$\begin{aligned}
c_j^* &= [\frac{\hat{q}_j^{-1}}{\frac{(\epsilon-1)C^{\frac{1}{\epsilon}}}{\epsilon}}]^{-\epsilon} \\
&= [\frac{\epsilon}{\epsilon-1}\frac{1}{\hat{q}_j}]^{-\epsilon}C \\
&= [\frac{1}{\frac{\epsilon}{\epsilon-1}\frac{1}{\hat{q}_j}}]^{\epsilon}C \\
&= [\frac{\epsilon-1}{\epsilon}\hat{q}_j]^{\epsilon}C
\end{aligned}$$

Which optimal quantity will correspond to the following equilibrium price for the intermediate good *j*

$$p_j = c_j^{*-\frac{1}{\epsilon}}C^{\frac{1}{\epsilon}} = \left(\frac{\epsilon-1}{\epsilon}\hat{q}_j]^{\epsilon}C\right)^{-\frac{1}{\epsilon}} C^{\frac{1}{\epsilon}}$$

then,

$$\begin{aligned}
p_j^* &= [\frac{\epsilon-1}{\epsilon}\hat{q}_j]^{-1}C^0 \\
&= \frac{\epsilon}{\epsilon-1}\frac{1}{\hat{q}_j}
\end{aligned}$$

The equilibrium profit is then given by

$$\begin{aligned}
\Pi_j^* &= \left(p_j^* - \hat{q}_j^{-1}\right)c_j^* \\
&= [\frac{\epsilon}{\epsilon-1}\frac{1}{\hat{q}_j} - \hat{q}_j^{-1}][\frac{\epsilon-1}{\epsilon}]^{\epsilon}C \\
&= \hat{q}_j^{-1}\hat{q}_j^{\epsilon}(\frac{1}{\epsilon-1})(\frac{\epsilon-1}{\epsilon})C \\
&= [\hat{q}_j\frac{\epsilon-1}{\epsilon}]^{\epsilon-1}\frac{1}{\epsilon}C \\
&= \frac{\hat{q}_j^{\epsilon-1}}{\epsilon-1}(\frac{\epsilon-1}{\epsilon})^{\epsilon}C
\end{aligned}$$

Appendix B

Unskilled wage and R&D cost

Appendix B1: Unskilled wage rate.

With the subscript t dropped out, the unskilled labour wage rate is given by

$$C = \left(\int_N c_j^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} = \left(\int_N ([\frac{\epsilon - 1}{\epsilon} \hat{q}_j]^{\epsilon})C)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$$

this leads to

$$\begin{aligned} \frac{C}{C} = 1 &= [\int_N (\frac{\epsilon - 1}{\epsilon} \hat{q}_j)^{\epsilon-1} dj]^{\frac{\epsilon}{\epsilon-1}} \\ &= (\frac{\epsilon - 1}{\epsilon})^{\epsilon} (\frac{1}{w^u})^{\epsilon} [\int_N q_j^{\epsilon-1} dj]^{\frac{\epsilon}{\epsilon-1}} \\ &= (\frac{\epsilon - 1}{\epsilon})^{\epsilon} (\frac{1}{w^u})^{\epsilon-1} Q^{\epsilon} \end{aligned}$$

This further leads to

$$\begin{aligned} (\frac{1}{w^u})^{\epsilon-1} &= \frac{1}{(\frac{\epsilon-1}{\epsilon})^{\epsilon} Q^{\epsilon}} \\ \Rightarrow (w^u)^{\epsilon} &= [(\frac{\epsilon - 1}{\epsilon})^{\epsilon} Q^{\epsilon}]^{\frac{1}{\epsilon}} \end{aligned}$$

Then,

$$w^u = \frac{\epsilon - 1}{\epsilon} Q$$

Appendix B2: Total R&D cost.

The total R&D cost will be the product of number of hired skilled labour and the skilled labour wage rate.We have:

$$C(x, \theta, \bar{q}_f) = w^s.h$$

We know that

$$x_{ft} = \theta_f^{\gamma} \bar{q}_{ft}^{\gamma} (\frac{h_{ft}}{n_{\ ft}})^{1-\gamma}$$

If I drop out the time subscript, I have:

$$\begin{aligned} \Rightarrow h_f &= x^{\frac{1}{1-\gamma}} n_f^{\frac{1-\gamma}{1-\gamma}} \theta_f^{-\frac{\gamma}{1-\gamma}} \bar{q}_f^{-\frac{\gamma}{1-\gamma}} \\ &= n_f \bar{q}_f^{-\frac{\gamma}{1-\gamma}} x^{\frac{1}{1-\gamma}} \theta^{-\frac{\gamma}{1-\gamma}} \\ &= n_f \bar{q}_f^{-\chi} G(x_f, \theta_f) \end{aligned}$$

then,

$$\begin{aligned} C(x, \theta, \bar{q}_f) &= w^s.h \\ &= w^s n_f \bar{q}_f^{-\chi} G(x_f, \theta_f) \end{aligned}$$

The R&D cost for an entrant is similarly determined.

Appendix C

R&D policy and productivity threshold for obsolescence

Appendix C1: R&D policy.

The R&D intensity value will be chosen by any given firm (either high-type or low-type) to maximize its expected net gain from doing research. The commitment to R&D has as objective the following programme:

$$max_{x^k\geq 0}\{ -w^s\phi + x^{entry}\mathbb{E}\Upsilon^k(\hat{q}(1+\lambda)) - w^s\bar{q}_f^{-\chi}G\left(x^{entry},\theta^{entry}\right)\} = 0 \qquad \text{(C.1)}$$

Here, $\mathbb{E}\Upsilon^{entry}(\hat{q}(1+\lambda))$ is the expected franchise value of the product line in which there the entrant successfully innovate and then draws the type-k upon entry. Then, we have

$$\frac{\partial(-w^s\phi + x^k\mathbb{E}\Upsilon^k(\hat{q}(1+\lambda)) - w^s\bar{q}_f^{-\chi}G\left(x^{entry},\theta^{entry}\right))}{\partial x}$$
$$= \mathbb{E}\Upsilon^k(\hat{q}(1+\lambda)) - \tilde{w}^s\bar{q}_f^{-\chi}\frac{1}{1-\gamma}x^{\frac{\gamma}{1-\gamma}}\theta^{-\frac{\gamma}{1-\gamma}}$$
$$\Rightarrow x_k = \theta^k\bar{q}_f[\frac{(1-\gamma)\mathbb{E}\Upsilon^k(\hat{q}(1+\lambda))}{\tilde{w}^s}]^{\frac{1-\gamma}{\gamma}}$$

Appendix C2: The obsolescence value of the relative productivity for a k-type firm.

At the limit value of the relative productivity, the derivative of the franchise value for the given product line equals zero. For a low-type firm, we have

$$\Upsilon_l\left(\hat{q}\right) = \frac{\Pi\hat{q}^{\epsilon-1}}{\Psi+(\epsilon-1)\,g}F_l\left(\frac{\Psi+(\epsilon-1)\,g}{g}\right) + \frac{\Omega^l-\tilde{w}^s\phi}{\Psi}F^l\left(\frac{\Psi}{g}\right)$$

$$\frac{\partial\Upsilon_l\left(\hat{q}\right)}{\partial\hat{q}}|_{\hat{q}=\hat{q}_{l,min}} = \frac{1}{g}[\Pi\hat{q}_{l,min}^{\epsilon-2} + \frac{(\Omega^l-\tilde{w}^s\phi)}{\hat{q}_{l,min}}]$$

$$\frac{\partial \Upsilon_l\left(\hat{q}\right)}{\partial \hat{q}}|_{\hat{q}=\hat{q}_{l,min}} = 0$$
$$\Rightarrow \hat{q}_{l,min} = [\frac{(\tilde{w}^s\phi) - \Omega^l}{\Pi}]^{\frac{1}{\epsilon-1}}$$

For a product line controlled by a high-type firm, we have

$$\Upsilon_h\left(\hat{q}\right) = \begin{cases} = \frac{\Pi\hat{q}^{\epsilon-1}}{\Psi+\nu+(\epsilon-1)g}F_h\left(\frac{\Psi+\nu+(\epsilon-1)g}{g}\right) - \frac{\tilde{w}^s\phi-\Omega^h}{\Psi+\nu}F_h\left(\frac{\Psi+\nu}{g}\right) & \textit{for } \hat{q}\in[\hat{q}_{h,min},\hat{q}_{l,min}] \\ = \left\{\frac{\Pi\hat{q}^{\epsilon-1}}{\Psi+\nu+(\epsilon-1)g}F_h\left(\frac{\Psi+\nu+(\epsilon-1)g}{g}\right) + \frac{\Omega^h-\tilde{w}^s\phi}{\Psi+\nu}F^h\left(\frac{\Psi+\nu}{g}\right)\right. & \\ +\frac{\Pi\hat{q}^{\epsilon-1}}{\Psi+(\epsilon-1)g}F_l\left(\frac{\Psi+(\epsilon-1)g}{g}\right) + \frac{\Omega^l-\tilde{w}^s\phi}{\Psi}F^l\left(\frac{\Psi}{g}\right) & \\ \left.-\frac{\Pi\hat{q}^{\epsilon-1}}{\Psi+\nu+(\epsilon-1)g}F_l\left(\frac{\Psi+\nu+(\epsilon-1)g}{g}\right) + \frac{\Omega^h-\tilde{w}^s\phi}{\Psi+\nu}F_l\left(\frac{\Psi+\nu}{g}\right)\right\} & \textit{for } \hat{q}\geq\hat{q}_{l,min} \end{cases}$$

$$\frac{\partial \Upsilon_h\left(\hat{q}\right)}{\partial \hat{q}}|_{\hat{q}=\hat{q}_{h,min}} = \frac{\Pi}{\Psi+\nu+(\epsilon-1)g}[(\epsilon-1)\hat{q}_{h,min}^{\epsilon-2} + \frac{\Psi+\nu}{g}\hat{q}_{h,min}^{\frac{\Psi+\nu+(\epsilon-1)g}{g}}\hat{q}_{h,min}] - \frac{\tilde{w}^s\phi-\Omega^h}{g}\hat{q}_{h,min}^{\frac{\Psi+\nu}{g}}\hat{q}_{h,min}^{-\frac{\Psi+\nu}{g}-1}$$

$$\frac{\partial \Upsilon_h\left(\hat{q}\right)}{\partial \hat{q}}|_{\hat{q}=\hat{q}_{h,min}} = \frac{\Pi\hat{q}_{h,min}^{\epsilon-2}}{\Psi+\nu+(\epsilon-1)g}[(\epsilon-1) + \frac{\Psi+\nu}{g}] - \frac{\tilde{w}^s\phi-\Omega^h}{g}\hat{q}_{h,min}^{-1}$$
$$= \frac{\Pi}{g}\hat{q}_{h,min}^{\epsilon-2} - \frac{\tilde{w}^s\phi-\Omega^h}{g}\hat{q}_{h,min}^{-1}$$

$$\frac{\partial \Upsilon_h\left(\hat{q}\right)}{\partial \hat{q}}|_{\hat{q}=\hat{q}_{h,min}} = 0$$
$$\Rightarrow \frac{\frac{\Pi}{g}}{\frac{\tilde{w}^s\phi-\Omega^h}{g}} = \frac{\hat{q}_{h,min}^{-1}}{\hat{q}_{h,min}^{\epsilon-2}}$$

$$\hat{q}_{h,min} = [\frac{(\tilde{w}^s\phi) - \Omega^h}{\Pi}]^{\frac{1}{\epsilon-1}}$$

Appendix D

Proof of the Proposition 1: The franchise value of low-type

A- For a low-type firm

The franchise value for a product line of relative productivity $\hat{q}$ controlled by a low-type firm is given by:

$$(r+\tau+\varphi)\,\Upsilon_l\,(\hat{q})-\frac{\partial\Upsilon^l\,(\hat{q})}{\partial\hat{q}}\frac{\partial\hat{q}}{\partial w^u}\frac{\partial w^u}{\partial t}=\left\{\Pi\hat{q}^{\epsilon-1}-\tilde{w}^s\phi+\omega^l\right\}\mathit{if}\;\hat{q}>\hat{q}_{l,min} \tag{D.1}$$

This is a linear first order differential equation non homogeneous. It can be equivalent to the following general form:

$$\Psi\Upsilon_l\,(\hat{q})-A\frac{\partial\Upsilon^l\,(\hat{q})}{\partial\hat{q}}=W \tag{D.2}$$

with $\Psi=(r+\tau+\varphi)$, $A=\frac{\partial\hat{q}}{\partial w^u}\frac{\partial w^u}{\partial t}$ and $W=\left\{\Pi\hat{q}^{\epsilon-1}-\tilde{w}^s\phi+\omega^l\right\}$

Given the specifications (3.21) and (3.4) the unskilled wage rate varies with the economy's productivity index Q, the relative productivity $\hat{q}$ will grow at the rate $-g$. Then we have

$$\begin{aligned}A&=\frac{\partial\hat{q}}{\partial w^u}\frac{\partial w^u}{\partial t}\\&=[\hat{q}(1-g)-\hat{q}]\\&=\hat{q}[(1-g)-1]\\&=-g\hat{q}\end{aligned}$$

(D.1) is then equivalent to

$$\begin{aligned}\Psi\Upsilon_l\,(\hat{q})-(-g\hat{q})\frac{\partial\Upsilon^l\,(\hat{q})}{\partial\hat{q}}&=W\\\frac{\Psi}{g}\hat{q}^{-1}\Upsilon_l\,(\hat{q})+\frac{\partial\Upsilon^l\,(\hat{q})}{\partial\hat{q}}&=\{\frac{\Pi}{g}\hat{q}^{\epsilon-2}-(\frac{\tilde{w}^s\phi-\omega^l}{g})\hat{q}^{-1}\}\\R_1\hat{q}^{-1}\Upsilon_l\,(\hat{q})+\frac{\partial\Upsilon^l\,(\hat{q})}{\partial\hat{q}}&=\left\{R_2\hat{q}^{\epsilon-2}-R_3\hat{q}^{-1}\right\}\end{aligned}$$

which delivers the following general form

$$U(\hat{q})\Upsilon_l(\hat{q}) + \frac{\partial \Upsilon^l(\hat{q})}{\partial \hat{q}} = W(\hat{q}) \tag{D.3}$$

With $U(\hat{q}) = R_1 = \frac{\Psi}{g}$, and $W(\hat{q}) = \left\{R_2\hat{q}^{\epsilon-2} - R_3\hat{q}^{-1}\right\} = \frac{\Pi}{g}\hat{q}^{\epsilon-2} - (\frac{\tilde{w}^s\phi-\omega^l}{g})\hat{q}^{-1}$

This differential equation (D.3) has as integrating factor $e^{\int U\partial\hat{q}}$ The general solution is then in the form

$$\begin{aligned}
\Upsilon_l(\hat{q}) &= e^{-\int R_1\hat{q}^{-1}\partial\hat{q}}[K + \int (R_2\hat{q}^{\epsilon-2} - R_3\hat{q}^{-1})e^{\int R_1\hat{q}^{-1}}\partial\hat{q}] \\
&= e^{-R_1\ln\hat{q}}[K + \int (R_2\hat{q}^{\epsilon-2} - R_3\hat{q}^{-1})e^{R_1\ln\hat{q}}\partial\hat{q}] \\
&= \hat{q}^{-R_1}[K + \int (R_2\hat{q}^{\epsilon-2+R_1} - R_3\hat{q}^{R_1-1})\partial\hat{q}] \\
&= \hat{q}^{-R_1}[K + \frac{1}{R_1+\epsilon-1}R_2\hat{q}^{R_1+\epsilon-1} - R_3\frac{1}{R_1}\hat{q}^{R_1}] \\
&= K\hat{q}^{-R_1} + \frac{R_2}{R_1+\epsilon-1}\hat{q}^{\epsilon-1} - \frac{R_3}{R_1}
\end{aligned}$$

Determination of the constant term K.

When $\hat{q}$ reaches $\hat{q}_{l,min}$, $\Upsilon_l(\hat{q}_{l,min}) = 0$

$$\begin{aligned}
\Upsilon_l(\hat{q}_{l,min}) &= 0 \\
\Rightarrow K\hat{q}_{l,min}^{-R_1} + \frac{R_2}{R_1+\epsilon-1}\hat{q}_{l,min}^{\epsilon-1} - \frac{R_3}{R_1} &= 0
\end{aligned}$$

$$K = \frac{R_3}{R_1}\hat{q}_{l,min}^{R_1} - \frac{R_2}{R_1+\epsilon-1}\hat{q}_{l,min}^{R_1+\epsilon-1}$$

Then we have,

$$\begin{aligned}
\Upsilon_l(\hat{q}) &= [\frac{R_3}{R_1}\hat{q}_{l,min}^{R_1} - \frac{R_2}{R_1+\epsilon-1}\hat{q}_{l,min}^{R_1+\epsilon-1}]\hat{q}^{-R_1} + \frac{R_2}{R_1+\epsilon-1}\hat{q}^{\epsilon-1} - \frac{R_3}{R_1} \\
&= [(\frac{\tilde{w}^s\phi - \Omega^l}{g})\frac{g}{\Psi}\hat{q}_{l,min}^{\frac{\Psi}{g}} - \frac{\frac{\Pi}{g}}{\frac{\Psi}{g}+\epsilon-1}\hat{q}_{l,min}^{\frac{\Psi}{g}+\epsilon-1}]\hat{q}^{-\frac{\Psi}{g}} + \frac{\frac{\Pi}{g}}{\frac{\Psi}{g}+\epsilon-1}\hat{q}^{\epsilon-1} - \frac{(\frac{\tilde{w}^s\phi-\Omega^l}{g})}{\frac{\Psi}{g}} \\
&= [-(\frac{\Omega^l - \tilde{w}^s\phi}{\Psi})\hat{q}_{l,min}^{\frac{\Psi}{g}} - \frac{\Pi}{\Psi+g(\epsilon-1)}\hat{q}_{l,min}^{\frac{\Psi}{g}+\epsilon-1}]\frac{1}{\hat{q}^{\frac{\Psi}{g}}} + \frac{\Pi}{\Psi+g(\epsilon-1)}\hat{q}^{\epsilon-1} + (\frac{\Omega^l - \tilde{w}^s\phi}{\Psi}) \\
&= (\frac{\Omega^l - \tilde{w}^s\phi}{\Psi})[1 - (\frac{\hat{q}_{l,min}}{\hat{q}})^{\frac{\Psi}{g}}] - \frac{\Pi}{\Psi+g(\epsilon-1)}\hat{q}_{l,min}^{\frac{\Psi}{g}+\epsilon-1}\frac{1}{\hat{q}^{\frac{\Psi}{g}}} + \frac{\Pi}{\Psi+g(\epsilon-1)}\hat{q}^{\epsilon-1} \\
&= (\frac{\Omega^l - \tilde{w}^s\phi}{\Psi})[1 - (\frac{\hat{q}_{l,min}}{\hat{q}})^{\frac{\Psi}{g}}] + \frac{\Pi}{\Psi+g(\epsilon-1)}\hat{q}^{\epsilon-1}[1 - (\frac{\hat{q}_{l,min}}{\hat{q}})^{\frac{\Psi}{g}+\epsilon-1}]
\end{aligned}$$

Defining

$$F_l(x) = [1 - (\frac{\hat{q}_{l,min}}{\hat{q}})^x]$$

we then have

$$\Upsilon_l\left(\hat{q}\right) = \left(\frac{\Omega^l - \tilde{w}^s\phi}{\Psi}\right)F_l\left(\frac{\Psi}{g}\right) + \frac{\Pi}{\Psi + g(\epsilon - 1)}\hat{q}^{\epsilon-1}F_l\left(\frac{\Psi}{g} + \epsilon - 1\right) \tag{D.4}$$

Appendix E

Proof of proposition 1: Franchise Value for a high-type

The franchise value for a high-type is given by the following differential equation:

$$(r+\tau+\varphi)\,\Upsilon_h(\hat{q}) - \frac{\partial \Upsilon^h(\hat{q})}{\partial \hat{q}}\frac{\partial \hat{q}}{\partial w^u}\frac{\partial w^u}{\partial t} = \left\{\Pi \hat{q}^{\epsilon-1} - \tilde{w}^s\phi + \Omega^h + \right. \tag{E.1}$$

$$\left.\nu[\mathbb{I}_{\hat{q}>\hat{q}_{l,min}}.\upsilon^l(\hat{q}) - \upsilon^h(\hat{q})]\right\} if\ \hat{q} > \hat{q}_{h,min} \tag{E.2}$$

$$\Upsilon_h(\hat{q}) = 0, Otherwise \tag{E.3}$$

Given that the high-type can transition to a low-type as long as $\hat{q} > \hat{q}_{l,min}$ since $\hat{q}_{l,min} > \hat{q}_{h,min}$, we can have 2 cases from (E.1)

first: $\hat{q}_{l,min} \geq \hat{q} \geq \hat{q}_{h,min}$ Since for $\hat{q} < \hat{q}_{l,min}$,$\Upsilon_l(\hat{q}) = 0$ we have the following equation:

$$(\Psi+\nu)\Upsilon_h(\hat{q}) - \frac{\partial \Upsilon^h(\hat{q})}{\partial \hat{q}}\frac{\partial \hat{q}}{\partial w^u}\frac{\partial w^u}{\partial t} = \left\{\Pi \hat{q}^{\epsilon-1} - \tilde{w}^s\phi + \Omega^h\right\}$$

Second:$\hat{q} \geq \hat{q}_{l,min}$ then we have the following equation

$$(r+\tau+\varphi)\,\Upsilon_h(\hat{q}) - \frac{\partial \Upsilon^h(\hat{q})}{\partial \hat{q}}\frac{\partial \hat{q}}{\partial w^u}\frac{\partial w^u}{\partial t} = \left\{\Pi \hat{q}^{\epsilon-1} - \tilde{w}^s\phi + \Omega^h + \nu[\Upsilon^l(\hat{q}) - \Upsilon^h(\hat{q})]\right\} \tag{E.4}$$

$$\Rightarrow (\Psi+\nu)\Upsilon_h(\hat{q}) - \frac{\partial \Upsilon^h(\hat{q})}{\partial \hat{q}}\frac{\partial \hat{q}}{\partial w^u}\frac{\partial w^u}{\partial t} = \left\{\Pi \hat{q}^{\epsilon-1} - \tilde{w}^s\phi + \Omega^h + \nu\Upsilon^l(\hat{q})\right\}$$

1- Let start with the first case: $\hat{q}_{l,min} \geq \hat{q} \geq \hat{q}_{h,min}$

$$(\Psi+\nu)\Upsilon_h(\hat{q}) - \frac{\partial \Upsilon^h(\hat{q})}{\partial \hat{q}}\frac{\partial \hat{q}}{\partial w^u}\frac{\partial w^u}{\partial t} = \left\{\Pi \hat{q}^{\epsilon-1} - \tilde{w}^s\phi + \Omega^h\right\}$$

$$\Rightarrow (\Psi+\nu)\Upsilon_h(\hat{q}) - \frac{\partial \Upsilon^h(\hat{q})}{(} - g\hat{q}) = \left\{\Pi \hat{q}^{\epsilon-1} - \tilde{w}^s\phi + \Omega^h\right\}$$

$$\Rightarrow (\frac{\Psi+\nu}{g})\hat{q}^{-1}\Upsilon_h(\hat{q}) + \frac{\partial \Upsilon^h(\hat{q})}{\partial \hat{q}} = (\frac{\Pi}{g})\hat{q}^{\epsilon-2} + (\frac{\Omega^h - \tilde{w}^s\phi}{g})\hat{q}^{-1}$$

Which leads to the following

$$K_1\hat{q}^{-1}\Upsilon_h(\hat{q}) + \frac{\partial\Upsilon^h(\hat{q})}{\partial\hat{q}} = K_2\hat{q}^{\epsilon-2} + K_3\hat{q}^{-1}$$

And the first order differential equation of the following general form

$$U(\hat{q})\Upsilon_h(\hat{q}) + \frac{\partial\Upsilon^h(\hat{q})}{\partial\hat{q}} = W(\hat{q}) \tag{E.5}$$

the general solution to (E.5) is

$$\begin{aligned}
\Upsilon_h(\hat{q}) &= e^{-\int U(\hat{q})\partial\hat{q}}[D + \int W(\hat{q})e^{\int U(\hat{q})}\partial\hat{q}] \\
&= e^{-\int K_1\hat{q}^{-1}\partial\hat{q}}[D + \int (K_2\hat{q}^{\epsilon-2} + K_3\hat{q}^{-1})e^{\int K_1\hat{q}^{-1}\partial\hat{q}}\partial\hat{q}] \\
&= e^{-K_1\ln\hat{q}}[D + \int (K_2\hat{q}^{\epsilon-2} + K_3\hat{q}^{-1})e^{K_1\ln\hat{q}}\partial\hat{q}] \\
&= \hat{q}^{-K_1}[D + \int (K_2\hat{q}^{\epsilon-2+K_1} + K_3\hat{q}^{-1+K_1})\partial\hat{q}] \\
&= \hat{q}^{-K_1}(D + \frac{K_2\hat{q}^{\epsilon-1+K_1}}{\epsilon-1+K_1} + \frac{K_3\hat{q}^{K_1}}{K_1})
\end{aligned}$$

$$\Upsilon_h(\hat{q}) = \hat{q}^{-K_1}(D + \frac{K_2\hat{q}^{\epsilon-1+K_1}}{\epsilon-1+K_1} + \frac{K_3\hat{q}^{K_1}}{K_1}) \tag{E.6}$$

For $\hat{q} = \hat{q}_{h,min}$,$\Upsilon_h(\hat{q}) = 0$ then, we have

$$\hat{q}_{h,min}^{-K_1}(D + \frac{K_2\hat{q}_{h,min}^{\epsilon-1+K_1}}{\epsilon-1+K_1} + \frac{K_3\hat{q}_{h,min}^{K_1}}{K_1}) = 0$$

$$\Rightarrow D = -(\frac{K_2\hat{q}_{h,min}^{\epsilon-1+K_1}}{\epsilon-1+K_1} + \frac{K_3\hat{q}_{h,min}^{K_1}}{K_1})$$

Replacing the constant D by this value into (E.6), we obtain

$$\begin{aligned}
\Upsilon_h(\hat{q}) &= \hat{q}^{-K_1}(\frac{K_2\hat{q}^{\epsilon-1+K_1}}{\epsilon-1+K_1} + \frac{K_3\hat{q}^{K_1}}{K_1} - \frac{K_2\hat{q}_{h,min}^{\epsilon-1+K_1}}{\epsilon-1+K_1} - \frac{K_3\hat{q}_{h,min}^{K_1}}{K_1}) \\
&= \hat{q}^{-\frac{\Psi+\nu}{g}}[\frac{\Pi\hat{q}^{\frac{\Psi+\nu}{g}+\epsilon-1}}{\Psi+\nu+g(\epsilon-1)} + (\frac{\Omega^h-\tilde{w}^s\phi}{\Psi+\nu})\hat{q}^{-\frac{\Psi+\nu}{g}} - \frac{\Pi\hat{q}_{h,min}^{\frac{\Psi+\nu}{g}+\epsilon-1}}{\Psi+\nu+g(\epsilon-1)} - (\frac{\Omega^h-\tilde{w}^s\phi}{\Psi+\nu})\hat{q}_{h,min}^{-\frac{\Psi+\nu}{g}}] \\
&= \frac{\Pi\hat{q}^{\epsilon-1}}{\Psi+\nu+g(\epsilon-1)} - \frac{\Pi}{\Psi+\nu+g(\epsilon-1)}\frac{\hat{q}_{h,min}^{\frac{\Psi+\nu+g(\epsilon-1)}{g}}}{\hat{q}^{\frac{\Psi+\nu}{g}}} + (\frac{\Omega^h-\tilde{w}^s\phi}{\Psi+\nu}) - \frac{\hat{q}_{h,min}^{\frac{\Psi+\nu}{g}}}{\hat{q}^{\frac{\Psi+\nu}{g}}}(\frac{\Omega^h-\tilde{w}^s\phi}{\Psi+\nu}) \\
&= \frac{\Pi\hat{q}^{\epsilon-1}}{\Psi+\nu+g(\epsilon-1)}[1 - (\frac{\hat{q}_{h,min}}{\hat{q}})^{\frac{\Psi+\nu+g(\epsilon-1)}{g}}] + (\frac{\Omega^h-\tilde{w}^s\phi}{\Psi+\nu})[1 - (\frac{\hat{q}_{h,min}}{\hat{q}})^{\frac{\Psi+\nu}{g}}]
\end{aligned}$$

Then, for $\hat{q} \in [\hat{q}_{h,min}, \hat{q}_{l,min}]$ we have

$$\Upsilon_h(\hat{q}) = \frac{\Pi\hat{q}^{\epsilon-1}}{\Psi+\nu+g(\epsilon-1)}F_l(\frac{\Psi+\nu+g(\epsilon-1)}{g}) + (\frac{\Omega^h - \tilde{w}^s\phi}{\Psi+\nu})F_l(\frac{\Psi+\nu}{g}) \tag{E.7}$$

2- Let turn back now to the second case:$\hat{q} \geq \hat{q}_{l,min}$
Then we have the following equation

$$\begin{aligned}(\Psi+\nu)\Upsilon^h(\hat{q}) - \frac{\partial\Upsilon^h(\hat{q})}{\partial\hat{q}}\frac{\partial\hat{q}}{\partial w^u}\frac{\partial w^u}{\partial t} &= \{\Pi\hat{q}^{\epsilon-1} - \tilde{w}^s\phi + \Omega^h + \nu\Upsilon^l(\hat{q})\} \\ (\Psi+\nu)\Upsilon^h(\hat{q}) - (-g\hat{q})\frac{\partial\Upsilon^h(\hat{q})}{\partial\hat{q}} &= \Pi\hat{q}^{\epsilon-1} - \tilde{w}^s\phi + \Omega^h + \nu\Upsilon^l(\hat{q}) \\ \frac{(\Psi+\nu)}{g}\hat{q}^{-1}\Upsilon^h(\hat{q}) + \frac{\partial\Upsilon^h(\hat{q})}{\partial\hat{q}} &= \frac{\Pi\hat{q}^{\epsilon-2}}{g} - \frac{(\tilde{w}^s\phi - \Omega^h)}{g}\hat{q}^{-1} + \frac{\nu\Upsilon^l(\hat{q})\hat{q}^{-1}}{g}\end{aligned} \tag{E.8}$$

From (D.4), we know that

$$\begin{aligned}\Upsilon^l(\hat{q}) &= (\frac{\Omega^l - \tilde{w}^s\phi}{\Psi})[1 - (\frac{\hat{q}_{l,min}}{\hat{q}})^{\frac{\Psi}{g}}] + \frac{\Pi}{\Psi+g(\epsilon-1)}\hat{q}^{\epsilon-1}[1 - (\frac{\hat{q}_{l,min}}{\hat{q}})^{\frac{\Psi}{g}+\epsilon-1}] \\ &= (\frac{\Omega^l - \tilde{w}^s\phi}{\Psi}) + [(\frac{\tilde{w}^s\phi - \Omega^l}{\Psi})\hat{q}_{l,min}^{\frac{\Psi}{g}} - \frac{\Pi}{\Psi+g(\epsilon-1)}\hat{q}_{l,min}^{\frac{\Psi+g(\epsilon-1)}{g}}]\hat{q}^{-\frac{\Psi}{g}} + \frac{\Pi}{\Psi+g(\epsilon-1)}\hat{q}^{\epsilon-1} \\ &= R_4 + R_5\hat{q}^{-\frac{\Psi}{g}} + R_6\hat{q}^{\epsilon-1}\end{aligned} \tag{E.9}$$

Inserting (E.9) into (E.8), we obtain

$$\begin{aligned}\frac{(\Psi+\nu)}{g}\hat{q}^{-1}\Upsilon^h(\hat{q}) + \frac{\partial\Upsilon^h(\hat{q})}{\partial\hat{q}} &= \frac{\Pi\hat{q}^{\epsilon-2}}{g} - \frac{(\tilde{w}^s\phi - \Omega^h)}{g}\hat{q}^{-1} + [\frac{\nu R_4}{g} + \frac{\nu R_5}{g}\hat{q}^{-\frac{\Psi}{g}} + \frac{\nu R_6}{g}\hat{q}^{\epsilon-1}]\hat{q}^{-1} \\ \frac{(\Psi+\nu)}{g}\hat{q}^{-1}\Upsilon^h(\hat{q}) + \frac{\partial\Upsilon^h(\hat{q})}{\partial\hat{q}} &= (\frac{\Pi+\nu R_6}{g})\hat{q}^{\epsilon-2} + (\frac{\Omega - \tilde{w}^s\phi + \nu R_4}{g})\hat{q}^{-1} + \frac{\nu R_5}{g}\hat{q}^{-\frac{\Psi+g}{g}}\end{aligned} \tag{E.10}$$

(E.10) is equivalent to

$$C_1\hat{q}^{-1}\Upsilon^h(\hat{q}) + \frac{\partial\Upsilon^h(\hat{q})}{\partial\hat{q}} = C_2\hat{q}^{\epsilon-2} + C_3\hat{q}^{-1} + C_4\hat{q}^{-\frac{\Psi+g}{g}} \tag{E.11}$$

which may be written under the above general form

$$U(\hat{q})\Upsilon_h(\hat{q}) + \frac{\partial\Upsilon^h(\hat{q})}{\partial\hat{q}} = W(\hat{q})$$

The general solution to this first order differential equation is

$$\begin{aligned}\Upsilon_h(\hat{q}) &= \hat{q}^{-C_1}[D + \int (C_2\hat{q}^{\epsilon-2} + C_3\hat{q}^{-1} + C_4\hat{q}^{-\frac{\Psi+g}{g}})\hat{q}^{C_1}\partial\hat{q}] \\ &= \hat{q}^{-C_1}[D + \frac{C_2\hat{q}^{C_1+\epsilon-1}}{C_1+\epsilon-1} + \frac{C_3\hat{q}^{C_1}}{C_1} + \frac{C_4\hat{q}^{C_1-\frac{\Psi}{g}}}{C_1-\frac{\Psi}{g}}]\end{aligned} \tag{E.12}$$

We know that at $\hat{q}_{l,min}$, (E.12) equals (E.7). This relationship helps us to determine the constant term D. We have

Replacing C_1, C_2, C_3 and C_4 by their corresponding value in (E.12) and considering it at $\hat{q}_{l,min}$, we obtain

$$\Upsilon_h(\hat{q}_{l,min}) = \hat{q}_{l,min}^{-\frac{\Psi+\nu}{g}}[D + (\frac{\tilde{w}^s-\Omega^l}{\Psi})\hat{q}_{l,min}^{\frac{\Psi+\nu}{g}} + \frac{\Psi(\Omega^h-\tilde{w}^s\phi)+\nu(\Omega^l-\tilde{w}^s)}{\Psi(\Psi+\nu)}\hat{q}_{l,min}^{\frac{\Psi+\nu}{g}}] \tag{E.13}$$

(E.7) at $\hat{q}_{l,min}$ is equivalent to:

$$\Upsilon_h(\hat{q}_{l,min}) = \frac{\Pi\hat{q}_{l,min}^{\epsilon-1}}{\Psi+\nu+g(\epsilon-1)}[1-(\frac{\hat{q}_{h,min}}{\hat{q}_{l,min}})^{\frac{\Psi+\nu+g(\epsilon-1)}{g}}] + (\frac{\Omega^h-\tilde{w}^s\phi}{\Psi+\nu})[1-(\frac{\hat{q}_{h,min}}{\hat{q}_{l,min}})^{\frac{\Psi+\nu}{g}}] \tag{E.14}$$

At $\hat{q}_{l,min}$, (E.13) equals (E.14) implies

$$\begin{aligned}D\hat{q}_{l,min}^{-\frac{\Psi+\nu}{g}} &+ (\frac{\tilde{w}^s-\Omega^l}{\Psi}) \\ &+ \frac{\Psi(\Omega^h-\tilde{w}^s\phi)+\nu(\Omega^l-\tilde{w}^s)}{\Psi(\Psi+\nu)} = \frac{\Pi\hat{q}_{l,min}^{\epsilon-1}}{\Psi+\nu+g(\epsilon-1)}[1-(\frac{\hat{q}_{h,min}}{\hat{q}_{l,min}})^{\frac{\Psi+\nu+g(\epsilon-1)}{g}}] \\ &\qquad + (\frac{\Omega^h-\tilde{w}^s\phi}{\Psi+\nu})[1-(\frac{\hat{q}_{h,min}}{\hat{q}_{l,min}})^{\frac{\Psi+\nu}{g}}]\end{aligned}$$

We can then pin down the constant D from the above equation

$$\begin{aligned}D = \frac{\Pi\hat{q}_{l,min}^{\frac{\Psi+\nu+(\epsilon-1)g}{g}}}{\Psi+\nu+(\epsilon-1)g} &- \frac{\Pi\hat{q}_{h,min}^{\frac{\Psi+\nu+(\epsilon-1)g}{g}}}{\Psi+\nu+(\epsilon-1)g} + (\frac{\Omega^h-\tilde{w}^s\phi}{\Psi+\nu})\hat{q}_{l,min}^{\frac{\Psi+\nu}{g}} - (\frac{\Omega^h-\tilde{w}^s\phi}{\Psi+\nu})\hat{q}_{h,min}^{\frac{\Psi+\nu}{g}} \\ &- (\frac{\Omega^h-\tilde{w}^s\phi}{\Psi})\hat{q}_{l,min}^{\frac{\Psi+\nu}{g}} - \frac{\Psi(\Omega^h-\tilde{w}^s\phi)+\nu(\Omega^l-\tilde{w}^s)}{\Psi(\Psi+\nu)}\hat{q}_{l,min}^{\frac{\Psi+\nu}{g}}\end{aligned}$$

$$\begin{aligned}\hat{q}^{-\frac{\Psi+\nu}{g}}D = \frac{\Pi\hat{q}^{\epsilon-1}}{\Psi+\nu+(\epsilon-1)g}(\frac{\hat{q}_{l,min}}{\hat{q}})^{\frac{\Psi+\nu+(\epsilon-1)g}{g}} &- \frac{\Pi\hat{q}^{\epsilon-1)}}{\Psi+\nu+(\epsilon-1)g}(\frac{\hat{q}_{h,min}}{\hat{q}})^{\frac{\Psi+\nu+(\epsilon-1)g}{g}} \\ + (\frac{\Omega^h-\tilde{w}^s\phi}{\Psi+\nu})(\frac{\hat{q_{l,min}}}{\hat{q}})^{\frac{\Psi+\nu}{g}} &- (\frac{\Omega^h-\tilde{w}^s\phi}{\Psi+\nu})(\frac{\hat{q_{h,min}}}{\hat{q}})^{\frac{\Psi+\nu}{g}} \\ - (\frac{\Omega^h-\tilde{w}^s\phi}{\Psi})(\frac{\hat{q}_{l,min}}{\hat{q}})^{\frac{\Psi+\nu}{g}} &- [\frac{\Psi(\Omega^h-\tilde{w}^s\phi)+\nu(\Omega^l-\tilde{w}^s)}{\Psi(\Psi+\nu)}](\frac{\hat{q}_{l,min}}{\hat{q}})^{\frac{\Psi+\nu}{g}}\end{aligned}$$

$$\hat{q}^{-\frac{\Psi+\nu}{g}}D = \frac{\Pi\hat{q}^{\epsilon-1}}{\Psi+\nu+(\epsilon-1)g}[1-(\frac{\hat{q}_{h,min}}{\hat{q}})^{\frac{\Psi+\nu+(\epsilon-1)g}{g}}] - \frac{\Pi\hat{q}^{\epsilon-1}}{\Psi+\nu+(\epsilon-1)g}[1-(\frac{\hat{q}_{l,min}}{\hat{q}})^{\frac{\Psi+\nu+(\epsilon-1)g}{g}}]$$
$$+(\frac{\Omega^h-\tilde{w}^s\phi}{\Psi+\nu})(\frac{\hat{q_{l,min}}}{\hat{q}})^{\frac{\Psi+\nu}{g}} - (\frac{\Omega^h-\tilde{w}^s\phi}{\Psi+\nu})(\frac{\hat{q_{h,min}}}{\hat{q}})^{\frac{\Psi+\nu}{g}} - (\frac{\Omega^h-\tilde{w}^s\phi}{\Psi})(\frac{\hat{q}_{l,min}}{\hat{q}})^{\frac{\Psi+\nu}{g}}$$
$$-[\frac{\Psi(\Omega^h-\tilde{w}^s\phi)+\nu(\Omega^l-\tilde{w}^s)}{\Psi(\Psi+\nu)}](\frac{\hat{q}_{l,min}}{\hat{q}})^{\frac{\Psi+\nu}{g}}$$

$$\hat{q}^{-\frac{\Psi+\nu}{g}}D = \frac{\Pi\hat{q}^{\epsilon-1}}{\Psi+\nu+(\epsilon-1)g}[1-(\frac{\hat{q}_{h,min}}{\hat{q}})^{\frac{\Psi+\nu+(\epsilon-1)g}{g}}] - \frac{\Pi\hat{q}^{\epsilon-1}}{\Psi+\nu+(\epsilon-1)g}[1-(\frac{\hat{q}_{l,min}}{\hat{q}})^{\frac{\Psi+\nu+(\epsilon-1)g}{g}}]$$
$$+(\frac{\Omega^h-\tilde{w}^s\phi}{\Psi+\nu})(\frac{\hat{q}_{l,min}}{\hat{q}})^{\frac{\Psi+\nu}{g}} - (\frac{\Omega^h-\tilde{w}^s\phi}{\Psi+\nu})(\frac{\hat{q}_{h,min}}{\hat{q}})^{\frac{\Psi+\nu}{g}} - (\frac{\Omega^h-\tilde{w}^s\phi}{\Psi})(\frac{\hat{q}_{l,min}}{\hat{q}})^{\frac{\Psi+\nu}{g}}$$
$$+(\frac{\Omega^l-\Omega^h}{\Psi+\nu}) - (\frac{\tilde{w}^s\phi-\Omega^h}{\Psi+\nu}) + (\frac{\tilde{w}^s\phi-\Omega^l}{\Psi+\nu}) - (\frac{\tilde{w}^s\phi-\Omega^l}{\Psi})(\frac{\hat{q}_{l,min}}{\hat{q}})^{\frac{\Psi+\nu}{g}} -$$
$$(\frac{\Psi(\Omega^h-\tilde{w}^s\phi)}{\Psi(\Psi+\nu)})(\frac{\hat{q_{l,min}}}{\hat{q}})^{\frac{\Psi+\nu}{g}} + (\frac{\nu(\tilde{w}^s\phi-\Omega^l)}{\Psi(\Psi+\nu)})(\frac{\hat{q_{l,min}}}{\hat{q}})^{\frac{\Psi+\nu}{g}}$$

$$\hat{q}^{-\frac{\Psi+\nu}{g}}D = \frac{\Pi\hat{q}^{\epsilon-1}}{\Psi+\nu+(\epsilon-1)g}[1-(\frac{\hat{q}_{h,min}}{\hat{q}})^{\frac{\Psi+\nu+(\epsilon-1)g}{g}}] - \frac{\Pi\hat{q}^{\epsilon-1}}{\Psi+\nu+(\epsilon-1)g}[1-(\frac{\hat{q}_{l,min}}{\hat{q}})^{\frac{\Psi+\nu+(\epsilon-1)g}{g}}]$$
$$+(\frac{\tilde{w}^s\phi-\Omega^l}{\Psi+\nu})[1-(\frac{\hat{q}_{l,min}}{\hat{q}})^{\frac{\Psi+\nu+(\epsilon-1)g}{g}}] - (\frac{\tilde{w}^s\phi-\Omega^h}{\Psi+\nu})[1-(\frac{\hat{q}_{h,min}}{\hat{q}})^{\frac{\Psi+\nu+(\epsilon-1)g}{g}}] + (\frac{\Omega^l-\Omega^h}{\Psi+\nu})$$

Now, we can replace $\hat{q}^{-\frac{\Psi+\nu}{g}}D$ by its expression into (E.12)

$$\Upsilon_h(\hat{q}) = \hat{q}^{-C_1}[D + \frac{C_2\hat{q}^{C_1+\epsilon-1}}{C_1+\epsilon-1} + \frac{C_3\hat{q}^{C_1}}{C_1} + \frac{C_4\hat{q}^{C_1-\frac{\Psi}{g}}}{C_1-\frac{\Psi}{g}}]$$
$$= \hat{q}^{-C_1}D + \frac{C_2\hat{q}^{\epsilon-1}}{C_1+\epsilon-1} + \frac{C_3}{C_1} + \frac{C_4\hat{q}^{-\frac{\Psi}{g}}}{C_1-\frac{\Psi}{g}}$$
$$= \frac{\frac{\Pi}{g}(\frac{\Psi+\nu+(\epsilon-1)g}{\Psi+(\epsilon-1)g})}{\frac{\Psi+\nu+(\epsilon-1)g}{g}}\hat{q}^{\epsilon-1} + \frac{\Psi(\Omega^h-\tilde{w}^s\phi)+\nu(\Omega^l-\tilde{w}^s\phi)}{\Psi g}\frac{g}{\Psi+\nu}$$
$$+[(\frac{\tilde{w}^s\phi-\Omega^l}{\Psi})\hat{q}_{l,min}^{\frac{\Psi}{g}} - \frac{\Pi\hat{q}_{l,min}^{\frac{\Psi+(\epsilon-1)g}{g}}}{\Psi+(\epsilon-1)g}]\hat{q}_{l,min}^{\frac{\Psi}{g}} + \hat{q}^{-C_1}D$$

Which leads to

$$\Upsilon_h\left(\hat{q}\right) = \frac{\Pi \hat{q}^{\epsilon-1}}{\Psi + (\epsilon-1)g}[1 - (\frac{\hat{q}_{l,min}}{\hat{q}})^{\frac{\Psi+(\epsilon-1)g}{g}}] + (\frac{\tilde{w}^s\phi - \Omega^l}{\Psi})(\frac{\hat{q}_{l,min}}{\hat{q}})^{\frac{\Psi}{g}} + \frac{\Psi(\Omega^h - \tilde{w}^s\phi) + \nu(\Omega^l - \tilde{w}^s\phi)}{\Psi(\Psi+\nu)} + \hat{q}^{-C_1} D$$

$$\begin{aligned}\Upsilon_h\left(\hat{q}\right) = {} & \frac{\Pi \hat{q}^{\epsilon-1}}{\Psi + (\epsilon-1)g}[1 - (\frac{\hat{q}_{l,min}}{\hat{q}})^{\frac{\Psi+(\epsilon-1)g}{g}}] + (\frac{\tilde{w}^s\phi - \Omega^l}{\Psi})(\frac{\hat{q}_{l,min}}{\hat{q}})^{\frac{\Psi}{g}} + \frac{\Psi(\Omega^h - \tilde{w}^s\phi) + \nu(\Omega^l - \tilde{w}^s\phi)}{\Psi(\Psi+\nu)} \\ & + \frac{\Pi \hat{q}^{\epsilon-1}}{\Psi + \nu + (\epsilon-1)g}[1 - (\frac{\hat{q}_{h,min}}{\hat{q}})^{\frac{\Psi+\nu+(\epsilon-1)g}{g}}] - \frac{\Pi \hat{q}^{\epsilon-1}}{\Psi + \nu + (\epsilon-1)g}[1 - (\frac{\hat{q}_{l,min}}{\hat{q}})^{\frac{\Psi+\nu+(\epsilon-1)g}{g}}] \\ & + (\frac{\tilde{w}^s\phi - \Omega^l}{\Psi+\nu})[1 - (\frac{\hat{q}_{l,min}}{\hat{q}})^{\frac{\Psi+\nu+(\epsilon-1)g}{g}}] - (\frac{\tilde{w}^s\phi - \Omega^h}{\Psi+\nu})[1 - (\frac{\hat{q}_{h,min}}{\hat{q}})^{\frac{\Psi+\nu+(\epsilon-1)g}{g}}] + (\frac{\Omega^l - \Omega^h}{\Psi+\nu})\end{aligned}$$

And we then have

$$\begin{aligned}\Upsilon_h\left(\hat{q}\right) = \Bigg\{ & \frac{\Pi \hat{q}^{\epsilon-1}}{\Psi + \nu + (\epsilon-1)\,g} F_h\left(\frac{\Psi+\nu+(\epsilon-1)\,g}{g}\right) + \frac{\Omega^h - \tilde{w}^s\phi}{\Psi+\nu} F^h\left(\frac{\Psi+\nu}{g}\right) \\ & + \frac{\Pi \hat{q}^{\epsilon-1}}{\Psi + (\epsilon-1)\,g} F_l\left(\frac{\Psi+(\epsilon-1)\,g}{g}\right) + \frac{\Omega^l - \tilde{w}^s\phi}{\Psi} F^l\left(\frac{\Psi}{g}\right) \\ & - \frac{\Pi \hat{q}^{\epsilon-1}}{\Psi + \nu + (\epsilon-1)\,g} F_l\left(\frac{\Psi+\nu+(\epsilon-1)\,g}{g}\right) + \frac{\Omega^h - \tilde{w}^s\phi}{\Psi+\nu} F_l\left(\frac{\Psi+\nu}{g}\right)\Bigg\}, \textit{for } \hat{q} \geq \hat{q}_{l,min}\end{aligned}$$

and the final expression given for the franchise value of owning a high-type product line:

$$\Upsilon_h\left(\hat{q}\right) = \begin{cases} = \frac{\Pi \hat{q}^{\epsilon-1}}{\Psi+\nu+(\epsilon-1)g} F_h\left(\frac{\Psi+\nu+(\epsilon-1)g}{g}\right) - \frac{\tilde{w}^s\phi - \Omega^h}{\Psi+\nu} F_h\left(\frac{\Psi+\nu}{g}\right) & \textit{for } \hat{q} \in [\hat{q}_{h,min}, \hat{q}_{l,min}] \\ = \Big\{ \frac{\Pi \hat{q}^{\epsilon-1}}{\Psi+\nu+(\epsilon-1)g} F_h\left(\frac{\Psi+\nu+(\epsilon-1)g}{g}\right) + \frac{\Omega^h - \tilde{w}^s\phi}{\Psi+\nu} F^h\left(\frac{\Psi+\nu}{g}\right) & \\ + \frac{\Pi \hat{q}^{\epsilon-1}}{\Psi+(\epsilon-1)g} F_l\left(\frac{\Psi+(\epsilon-1)g}{g}\right) + \frac{\Omega^l - \tilde{w}^s\phi}{\Psi} F^l\left(\frac{\Psi}{g}\right) & \\ - \frac{\Pi \hat{q}^{\epsilon-1}}{\Psi+\nu+(\epsilon-1)g} F_l\left(\frac{\Psi+\nu+(\epsilon-1)g}{g}\right) + \frac{\Omega^h - \tilde{w}^s\phi}{\Psi+\nu} F_l\left(\frac{\Psi+\nu}{g}\right) \Big\} & \textit{for } \hat{q} \geq \hat{q}_{l,min} \end{cases}$$

Appendix F

Proof of Lemma 3: Stationary Equilibrium productivity Distribution

1- Let consider the product lines controlled by high-type firms.
In the stationary equilibrium,inflows and the outflows into different parts of the distribution are equal. That is, with $F_h(\hat{q})$ as the stationary equilibrium productivity distributions of product lines controlled by high-type, and considering a time interval Δt, $\Phi^h F_h(\hat{q},t) = \Phi^h F_h(\hat{q}, t+\Delta t)$

$$\Phi^h F_h(\hat{q}) = \begin{cases} = \Phi(1-(\tau+\nu+\rho)\Delta t)[F_h(\hat{q}(1+g\Delta t)) - F_h(\hat{q}_{h,min}(1+g\Delta t))] \\ +(\Phi x^h + \alpha X^{entry})\Delta t[\Phi^h F_h(\hat{q}\frac{(1+g\Delta t)}{1+\lambda}) + \Phi^l F_l(\hat{q}\frac{(1+g\Delta t)}{1+\lambda}) \\ +\Phi^{np} F(\hat{q})] \textit{for } \hat{q} > \hat{q}_{h,min}(1+\lambda) \\ = \Phi(1-(\tau+\nu+\rho)\Delta t)[F_h(\hat{q}(1+g\Delta t)) \\ -F_h(\hat{q}_{h,min}(1+g\Delta t))] \textit{for } \hat{q}_{h,min}(1+g\Delta t) < \hat{q} \leq \hat{q}_{h,min}(1+\lambda) \end{cases}$$

A- Let consider the first option, that is when $\hat{q} > \hat{q}_{h,min}(1+\lambda)$

$$\begin{aligned} \Phi^h F_h(\hat{q}) - \Phi^h F_h(\hat{q}(1+g\Delta t)) = \Phi(1-(\tau+\nu+\rho)\Delta t)[F_h(\hat{q}(1+g\Delta t)) - F_h(\hat{q}_{h,min}(1+g\Delta t))] \\ + (\Phi x^h + \alpha X^{entry})\Delta t[\Phi^h F_h(\hat{q}\frac{(1+g\Delta t)}{1+\lambda}) + \Phi^l F_l(\hat{q}\frac{(1+g\Delta t)}{1+\lambda}) \\ + \Phi^{np} F(\hat{q})] - \Phi^h F_h(\hat{q}(1+g\Delta t)) \end{aligned}$$

It follows that

$$\begin{aligned} \Phi^h F_h(\hat{q}(1+g\Delta t)) - \Phi^h F_h(\hat{q}) = \Phi^h F_h(\hat{q}(1+g\Delta t)) - \Phi^h F_h(\hat{q}(1+g\Delta t)) \\ + \Phi^h(\tau+\nu+\rho)\Delta t) F_h(\hat{q}(1+g\Delta t)) + \Phi^h(1-(\tau+\nu+\rho)\Delta t)) F_h(\hat{q}_{h,min}(1+g\Delta t)) \\ - (\Phi x^h + \alpha X^{entry})\Delta t[\Phi^h F_h(\hat{q}\frac{(1+g\Delta t)}{1+\lambda}) + \Phi^l F_l(\hat{q}\frac{(1+g\Delta t)}{1+\lambda}) \\ + \Phi^{np} F(\hat{q})] \end{aligned}$$

Then

$$\frac{\Phi^h[F_h(\hat{q}(1+g\Delta t)) - F_h(\hat{q}) - F_h(\hat{q}_{h,min}(1+g\Delta t))]}{\Delta t} = \Phi^h(\tau+\nu+\rho))F_h(\hat{q}(1+g\Delta t))$$
$$- \Phi^h(\tau+\nu+\rho))F_h(\hat{q}_{h,min}(1+g\Delta t))$$
$$- (\Phi x^h + \alpha X^{entry})\Delta t[\Phi^h F_h(\hat{q}\frac{(1+g\Delta t)}{1+\lambda}) + \Phi^l F_l(\hat{q}\frac{(1+g\Delta t)}{1+\lambda}) + \Phi^{np}F(\hat{q})]$$

$$\frac{[F_h(\hat{q}(1+g\Delta t)) - F_h(\hat{q}) - F_h(\hat{q}_{h,min}(1+g\Delta t))]}{\Delta t} = (\tau+\nu+\rho))[F_h(\hat{q}(1+g\Delta t))$$
$$- F_h(\hat{q}_{h,min}(1+g\Delta t))]$$
$$- \frac{(\Phi x^h + \alpha X^{entry})}{\Phi^h}[\Phi^h F_h(\hat{q}\frac{(1+g\Delta t)}{1+\lambda}) + \Phi^l F_l(\hat{q}\frac{(1+g\Delta t)}{1+\lambda}) + \Phi^{np}F(\hat{q})] \quad \text{(F.1)}$$

Let consider the LHS of (F.1) and we have

$$\lim_{\Delta t \to 0} \frac{[F_h(\hat{q}(1+g\Delta t)) - F_h(\hat{q}) - F_h(\hat{q}_{h,min}(1+g\Delta t))]}{\Delta t} = \frac{\partial F_h(\hat{q}(1+g\Delta t))}{\partial \Delta t} - \frac{\partial F_h(\hat{q})}{\partial \Delta t}$$
$$- \frac{\partial F_h(\hat{q}_{h,min}(1+g\Delta t))}{\partial \Delta t}$$
$$= g\hat{q}f_h(\hat{q}) - 0 - g\hat{q}_{h,min}f_h(\hat{q}_{h,min})$$
$$= g\hat{q}f_h(\hat{q}) - g\hat{q}_{h,min}f_h(\hat{q}_{h,min})$$

For the RHS of (F.1), we have:

$$\lim_{\Delta t \to 0}(RHS) = (\tau+\nu+\rho))[F_h(\hat{q}) - F_h(\hat{q}_{h,min})]$$
$$- \frac{(\Phi x^h + \alpha X^{entry})}{\Phi^h}[\Phi^h F_h(\frac{\hat{q}}{1+\lambda}) + \Phi^l F_l(\frac{\hat{q}}{1+\lambda}) + \Phi^{np}F(\hat{q})]$$

we then have the following equation for $\hat{q} > \hat{q}_{h,min}(1+\lambda)$

$$g\hat{q}f_h(\hat{q}) - g\hat{q}_{h,min}f_h(\hat{q}_{h,min}) = (\tau+\nu+\rho))[F_h(\hat{q}) - F_h(\hat{q}_{h,min})]$$
$$- \frac{(\Phi x^h + \alpha X^{entry})}{\Phi^h}[\Phi^h F_h(\frac{\hat{q}}{1+\lambda}) + \Phi^l F_l(\frac{\hat{q}}{1+\lambda}) + \Phi^{np}F(\hat{q})]$$

B- Let consider the second option, that is $\hat{q}_{h,min}(1+\lambda) < \hat{q} \leq \hat{q}_{h,min}(1+g\Delta t)$

$$\frac{\Phi^h F_h(\hat{q}(1+g\Delta t)) - \Phi^h F_h(\hat{q})}{\Delta t} = \Phi^h(\tau+\nu+\rho)[F_h(\hat{q}(1+g\Delta t))$$
$$- F_h(\hat{q}_{h,min}(1+g\Delta t))] + \frac{\Phi^h F_h(\hat{q}_{h,min})}{\Delta t}\Delta t$$

$$\lim_{\Delta t \to 0} \frac{\Phi^h F_h(\hat{q}(1+g\Delta t)) - \Phi^h F_h(\hat{q}) - \Phi^h F_h(\hat{q}_{h,min})}{\Delta t} = \lim_{\Delta t \to 0} (\Phi^h(\tau+\nu+\rho)[F_h(\hat{q}(1+g\Delta t)) - F_h(\hat{q}_{h,min}(1+g\Delta t))])$$

which leads to

$$g\hat{q}f_h(\hat{q}) - g\hat{q}_{h,min}f_h(\hat{q}_{h,min}) = (\tau+\nu+\rho)[F_h(\hat{q}) - F_h(\hat{q}_{h,min})] \quad \textit{for } \hat{q}_{h,min}(1+\lambda) < \hat{q} \leq \hat{q}_{h,min}(1+g\Delta t)$$

To sump up, we then have the following equations that are satisfied by the stationary equilibrium productivity distributions of active product lines of high-type.

$$g\hat{q}f_h(\hat{q}) = \begin{cases} = g\hat{q}_{h,min}f_h(\hat{q}_{h,min}) + (\tau+\nu+\rho))[F_h(\hat{q}) - F_h(\hat{q}_{h,min})] \\ -\frac{(\Phi x^h + \alpha X^{entry})}{\Phi^h}[\Phi^h F_h(\frac{\hat{q}}{1+\lambda}) + \Phi^l F_l(\frac{\hat{q}}{1+\lambda}) + \Phi^{np}F(\hat{q})] \\ \textit{for } \hat{q} > \hat{q}_{h,min}(1+\lambda) \\ = g\hat{q}_{h,min}f_h(\hat{q}_{h,min}) + (\tau+\nu+\rho)[F_h(\hat{q}) - F_h(\hat{q}_{h,min})] \\ \textit{for } \hat{q}_{h,min}(1+\lambda) < \hat{q} \leq \hat{q}_{h,min}(1+g\Delta t) \end{cases}$$

2- For the low-type, similar steps are followed and lead to the equations at 3.23

Appendix G

Aggregate output growth rate

We know from (3.5) that the economy's productivity index is given by

$$Q = \left(\int_{\mathcal{N}_t} q_j^{\epsilon-1} dj \right)^{\frac{1}{\epsilon-1}}$$

with $\mathcal{N}_t = \mathcal{N}_t^h + \mathcal{N}_t^l$

Let us define

$$\tilde{Q}_t \equiv Q^{\epsilon-1}$$

Then we have

$$\begin{aligned}
\tilde{Q}_t &= \left(\int_{\mathcal{N}_t} q_j^{\epsilon-1} dj \right)^{\frac{1}{\epsilon-1}(\epsilon-1)} \\
&= \int_{\mathcal{N}_t} q_j^{\epsilon-1} dj \\
&= \int_{\mathcal{N}_t^h} q_j^{\epsilon-1} dj + \int_{\mathcal{N}_t^l} q_j^{\epsilon-1} dj
\end{aligned}$$

If we consider a short time Δt, the productivity index of product lines owned by high-type firms will become $\tilde{Q}_{t+\Delta t}^h$ which is given by:

$$\begin{aligned}
\tilde{Q}_{t+\Delta t}^h = \int_{\mathcal{N}_t^h} &\{ (x^h + \alpha X^{entry}) \Delta t [(1+\lambda) q_{jt}]^{\epsilon-1} \\
&+ (1 - \tau \Delta t - \nu \Delta t - \rho \Delta t)[1 - F_h((1 + g\Delta t)\hat{q}_{h,min})] q_{jt}^{\epsilon-1} \} dj \\
&+ \int_{\mathcal{N}_t^l} (x^h + \alpha X^{entry}) \Delta t [(1+\lambda) q_{jt}]^{\epsilon-1} dj \\
&+ \int_{\mathcal{N}_t^{np}} (x^h + \alpha X^{entry} + \varrho \Phi^h) \Delta t \mathbb{E} q_{t+\Delta t}^{\epsilon-1} dj \qquad \text{(G.1)}
\end{aligned}$$

Intuitive explanation of the RHS of (G.1)

First line: For the short time period Δt, high-type incumbents and high-type entrants can improve upon existing product lines owned yet by high-type incumbents, this at the total rate $(x^h + \alpha X^{entry})$ and the productivity goes from $q_{jt} \rightarrow \Delta t(1+\lambda) q_{jt}$

Second line: For the short time period Δt, product lines own by high-type will undergo an overall destructive shock of total rate $(\tau - \nu - \rho)\Delta t$. This line gives the productivity of high-type incumbent product lines that would have survived the global shock at the end of the length of time Δt. The relative productivity of such product line $\hat{q}$ is $\hat{q} \geq (1 + g\Delta)\hat{q}_{h,min}$ and

$$Pr[\hat{q} > (1 + g\Delta)\hat{q}_{h,min}] = 1 - F_h((1 + g\Delta t)\hat{q}_{h,min})$$

The third line: For the short time period Δt, this gives the high-type productivity increase due to high-type entrant and high-type incumbent improvements upon product lines yet owned by low-type incumbents.
The fourth line:high-type incumbent and high-type entrant improvements upon inactive product lines and also a non-R&D related positive productive shock in high-type firms on inactive product lines.

The growth rate of productivity of product lines own by high-type firms g^h is given by

$$\begin{aligned} g^h = \frac{\frac{\partial \tilde{Q}_t^h}{\partial t}}{\tilde{Q}_t^h} \\ &= \frac{\lim_{\Delta t \to 0} \frac{\tilde{Q}^h_{t+\Delta t} - \tilde{Q}^h_t}{\tilde{Q}^h_t \Delta t}}{\tilde{Q}^h_t} \\ &= \lim_{\Delta t \to 0} \frac{\tilde{Q}^h_{t+\Delta t} - \tilde{Q}^h_t}{\tilde{Q}^h_t \Delta t} \end{aligned}$$

Then we have

$$\begin{aligned} \tilde{Q}^h_{t+\Delta t} - \tilde{Q}^h_t = \int_{N_t^h} (x^h + \alpha X^{entry})\Delta t[(1+\lambda)]^{\epsilon-1} q_{jt}^{\epsilon-1} dj \\ + \int_{N_t^h} [1 - F_h((1 + g\Delta t)\hat{q}_{h,min})] q_{jt}^{\epsilon-1} dj \\ + \int_{N_t^h} \Delta t(\tau + \nu + \phi) F_h((1 + g\Delta t)\hat{q}_{h,min}) q_{jt}^{\epsilon-1} dj \\ - \int_{N_t^h} (\tau + \nu + \phi)\Delta t q_{jt}^{\epsilon-1} dj + \int_{N_t^l} (x^h + \alpha X^{entry})\Delta t[(1+\lambda)]^{\epsilon-1} q_{jt}^{\epsilon-1} dj \\ + \int_{N_t^{np}} (x^h + \alpha X^{entry} + \varrho \Phi^h)\Delta t \mathbb{E} q_{t+\Delta t}^{\epsilon-1} dj - \tilde{Q}^h_t \end{aligned}$$

which leads to

$$\begin{aligned}\tilde{Q}^h_{t+\Delta t} - \tilde{Q}^h_t = (x^h + \alpha X^{entry})\Delta t[(1+\lambda)]^{\epsilon-1}\tilde{Q}^h_t + \tilde{Q}^h_t \\ - F_h((1+g\Delta t)\hat{q}_{h,min})[1-\Delta t(\tau+\nu+\phi)]\tilde{Q}^h_t - \Delta t(\tau+\nu+\phi)\tilde{Q}^h_t \\ + (x^h + \alpha X^{entry})\tilde{Q}^l_t \Delta t[(1+\lambda)]^{\epsilon-1} \\ + (x^h + \alpha X^{entry} - \varrho\Phi^h)\Delta t \Phi^{np}\mathbb{E}q^{\epsilon-1}_{t+\Delta t} - \tilde{Q}^h_t\end{aligned}$$

Then we can have

$$\begin{aligned}\frac{\tilde{Q}^h_{t+\Delta t} - \tilde{Q}^h_t}{\tilde{Q}^h_t \Delta t} = (x^h + \alpha X^{entry})[(1+\lambda)]^{\epsilon-1} \\ - \frac{F_h((1+g\Delta t)\hat{q}_{h,min})[1-\Delta t(\tau+\nu+\phi)]}{\Delta t} - (\tau+\nu+\phi) \\ + (x^h + \alpha X^{entry})[(1+\lambda)]^{\epsilon-1}\frac{\tilde{Q}^l_t}{\tilde{Q}^h_t} \\ + (x^h + \alpha X^{entry} - \varrho\Phi^h)\frac{\Phi^{np}\mathbb{E}q^{\epsilon-1}_{t+\Delta t}}{\tilde{Q}^h_t}\end{aligned}$$

$$\begin{aligned}\lim_{\Delta t\to 0}\frac{\tilde{Q}^h_{t+\Delta t} - \tilde{Q}^h_t}{\tilde{Q}^h_t \Delta t} = (x^h + \alpha X^{entry})[(1+\lambda)]^{\epsilon-1} \\ - \lim_{\Delta t\to 0}\frac{F_h((1+g\Delta t)\hat{q}_{h,min})[1-\Delta t(\tau+\nu+\phi)]}{\Delta t} \\ - (\tau+\nu+\phi) + (x^h + \alpha X^{entry})[(1+\lambda)]^{\epsilon-1}\frac{\tilde{Q}^l_t}{\tilde{Q}^h_t} \\ + (x^h + \alpha X^{entry} - \varrho\Phi^h)\frac{\Phi^{np}\mathbb{E}q^{\epsilon-1}_{t+}}{\tilde{Q}^h_t}\end{aligned}$$

$$\begin{aligned}\lim_{\Delta t\to 0}\frac{\tilde{Q}^h_{t+\Delta t} - \tilde{Q}^h_t}{\tilde{Q}^h_t \Delta t} = (x^h + \alpha X^{entry})[(1+\lambda)]^{\epsilon-1} \\ - g\hat{q}_{h,min}f_h(\hat{q}_{h,min}) - (\tau+\nu+\phi) \\ + (x^h + \alpha X^{entry})[(1+\lambda)]^{\epsilon-1}\frac{\tilde{Q}^l_t}{\tilde{Q}^h_t} \\ + (x^h + \alpha X^{entry} - \varrho\Phi^h)\frac{\Phi^{np}\mathbb{E}q^{\epsilon-1}_t}{\tilde{Q}^h_t}\end{aligned}$$

Then,we have

$$g^h = \big\{(x^h + \alpha X^{entry})[(1+\lambda)^{\epsilon-1}(1+\frac{\tilde{Q}^l_t}{\tilde{Q}^h_t}) + \frac{\Phi^{np}\mathbb{E}q_t^{\epsilon-1}}{\tilde{Q}^h_t}]$$
$$- (\tau+\nu+\phi) + \varrho\Phi^{np}\frac{\Phi^{np}\mathbb{E}q_t^{\epsilon-1}}{\tilde{Q}^h_t} - g\hat{q}_{h,min}f_h(\hat{q}_{h,min})\big\} \quad \text{(G.2)}$$

For product lines owned by low-type firms, we proceed via similar steps and we obtain

$$g^l = \big\{(x^h + (1-\alpha) X^{entry})[(1+\lambda)^{\epsilon-1}(1+\frac{\tilde{Q}^h_t}{\tilde{Q}^l_t}) + \frac{\Phi^{np}\mathbb{E}q_t^{\epsilon-1}}{\tilde{Q}^l_t}]$$
$$- (\tau+\phi) + \nu[1-F_l(\hat{q}_{l,min})]\frac{\tilde{Q}^h_t}{\tilde{Q}^l_t}$$
$$+ \varrho\Phi^{l}\frac{\Phi^{np}\mathbb{E}q_t^{\epsilon-1}}{\tilde{Q}^l_t} - g\hat{q}_{l,min}f_h(\hat{q}_{l,min})\big\} \quad \text{(G.3)}$$

In the stationary equilibrium, the ration $\frac{\tilde{Q}^h_t}{\tilde{Q}^l_t}$ remains constant, which means that $\tilde{Q}_l$ and $\tilde{Q}_h$ grow at the same rate. In other words, at the stationary equilibrium, $g^l = g^h$

At the same time, we know that

$$\tilde{Q}_t = Q_t^{\epsilon-1}$$
$$\Rightarrow Q_t = \tilde{Q}_t^{\frac{1}{\epsilon-1}}$$

For g less than 10 percent, we have

$$\begin{aligned} g &= \ln Q_{t+1} - \ln Q_t \\ &= \ln \tilde{Q}_{t+1}^{\frac{1}{\epsilon-1}} - \ln \tilde{Q}_t^{\frac{1}{\epsilon-1}} \\ &= \frac{1}{\epsilon-1}(\ln \tilde{Q}_{t+1} - \ln \tilde{Q}_t) \\ &= \frac{g^h}{\epsilon-1} = \frac{g^l}{\epsilon-1} \end{aligned}$$

Using the productivity index of product lines owned by high-type firms, the growth rate of the economy is then given by:

$$g = \frac{g^h}{\epsilon-1} = \frac{(x^h + \alpha X^{entry})[(1+\lambda)^{\epsilon-1}(1+\frac{\tilde{Q}^l_t}{\tilde{Q}^h_t}) + \frac{\Phi^{np}\mathbb{E}q_t^{\epsilon-1}}{\tilde{Q}^h_t}]}{\epsilon-1}$$
$$+ \frac{\varrho\Phi^{np}\frac{\Phi^{np}\mathbb{E}q_t^{\epsilon-1}}{\tilde{Q}^h_t} - (\tau+\nu+\phi) - g\hat{q}_{h,min}f_h(\hat{q}_{h,min})}{\epsilon-1}$$

$$\Rightarrow g(1+\frac{\hat{q}_{h,min}f_h(\hat{q}_{h,min})}{\epsilon-1}) = \frac{(x^h+\alpha X^{entry})[(1+\lambda)^{\epsilon-1}(1+\frac{\tilde{Q}_t^l}{\tilde{Q}_t^h})+\frac{\Phi^{np}\mathbb{E}q_t^{\epsilon-1}}{\tilde{Q}_t^h}]}{\epsilon-1} + \frac{\varrho\Phi^{np}\frac{\Phi^{np}\mathbb{E}q_t^{\epsilon-1}}{\tilde{Q}_t^h}-(\tau+\nu+\phi)}{\epsilon-1}$$

then,

$$g = \frac{(x^h+\alpha X^{entry})[(1+\lambda)^{\epsilon-1}(1+\frac{\tilde{Q}_t^l}{\tilde{Q}_t^h})+\frac{\Phi^{np}\mathbb{E}q_t^{\epsilon-1}}{\tilde{Q}_t^h}]+\varrho\Phi^h\frac{\Phi^{np}\mathbb{E}q_t^{\epsilon-1}}{\tilde{Q}_t^h}-(\tau+\nu+\phi)}{(\epsilon-1)+\hat{q}_{h,min}f_h(\hat{q}_{h,min})}$$

Defining $\Gamma = \frac{\tilde{Q}_t^h}{\tilde{Q}_t^l}$ and $\kappa_k = \frac{\Phi^{np}\mathbb{E}q_t^{\epsilon-1}}{\tilde{Q}_t^k}$ we finally have

$$g = \frac{(x^h+\alpha X^{entry})[(1+\lambda)^{\epsilon-1}(1+\frac{1}{\Gamma})+\kappa_h]+\varrho\Phi^h\kappa_h-(\tau+\nu+\phi)}{(\epsilon-1)+\hat{q}_{h,min}f_h(\hat{q}_{h,min})}$$

In such a way that Γ is solution to the equation $g^l = g^h$, that is

$$\begin{aligned}(x^h+\alpha X^{entry})[(1+\lambda)^{\epsilon-1}(1+\frac{1}{\Gamma})+\kappa_h]+\varrho\kappa^h[\Phi^h-\Phi^l\Gamma] &= \nu[1+[1-F_h(\hat{q}_{l,min})]\Gamma] \\ &+ (x^l+\alpha X^{entry})[(1+\lambda)^{\epsilon-1}(1+\frac{1}{\Gamma})+\kappa_l] \\ &+ g[\hat{q}_{h,min}f_h(\hat{q}_{h,min})-\hat{q}_{h,min}f_l(\hat{q}_{l,min})]\end{aligned}$$

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